

Circe1 (internal Version 2.2): Beam Spectra for Simulating Linear Collider Physics*

Thorsten Ohl[†]

University of Würzburg
Emil-Hilb-Weg 22
D-97089 Würzburg
Germany

IKDA 96/13-rev
hep-ph/9607454-rev

prelude.nw 5455 2014-03-12 14:26:30Z ohl
circe1.nw 5607 2014-03-31 09:28:28Z ohl
minuit.nw 5455 2014-03-12 14:26:30Z ohl
postlude.nw 5455 2014-03-12 14:26:30Z ohl
July 1996 (expanded March 2014)

Abstract

I describe parameterizations of realistic e^\pm - and γ -beam spectra at future linear e^+e^- -colliders. Emphasis is put on simplicity and reproducibility of the parameterizations, supporting reproducible physics simulations. The parameterizations are implemented in a library of distribution functions and event generators.

Contents

1	Introduction	4
2	Parameters	5
3	Usage	11
3.1	Distributions	11
3.1.1	Example	14
3.2	Generators	19
3.2.1	Example	20
3.2.2	Event Generators	21

*Supported by Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie, Germany.

[†]e-mail: ohl@physik.uni-wuerzburg.de

4	Technical Notes	22
5	Parameterizations	23
5.1	Version 1	23
5.1.1	Fitting	27
5.1.2	Generators	29
5.2	Future Versions	29
6	Implementation of circe1	29
6.1	Symbolic Constants	30
6.2	Distributions	31
6.2.1	Version 1	31
6.2.2	Version 2	50
6.2.3	Versions 3 and 4	51
6.2.4	Version 5	53
6.2.5	Version 6	54
6.2.6	Version 7	57
6.2.7	Version 8	61
6.2.8	Version 9	63
6.2.9	Version 10	69
6.3	Special Functions	71
6.4	Non-Singular Distributions	73
6.5	Generators	78
6.5.1	Random-Number Generator	78
6.5.2	Version 1	80
6.5.3	Version 2	83
6.5.4	Version 3 and 4	83
6.6	Utilities	83
6.7	Examples	87
6.7.1	Distributions	87
6.7.2	Library functions	88
6.7.3	Generators	91
6.8	Dumping Parameters	91
7	Fitting	93
7.1	Version 1: Factorized Beta Distributions	93
7.2	Experimental	110
7.2.1	Quasi One Dimensional	110
7.2.2	Quasi Two Dimensional	115
7.3	Version 2	118
8	Conclusions	118
A	Literate Programming	126
A.1	Paradigm	126
A.2	Practice	126
B	Fortran Name Space	127
C	Updates	127

Program Summary:

- **Title of program:** Circe1 (March 2014)
- **Program obtainable** by anonymous ftp from the host `crunch.ikp.physik.th-darmstadt.de` in the directory `pub/ohl/circe`.
- **Licensing provisions:** Free software under the GNU General Public License.
- **Programming language used:** Fortran77 originally, transferred to Fortran90
- **Number of program lines in distributed program, including test data, etc.:** ≈ 1100 (excluding comments)
- **Computer/Operating System:** Any with a Fortran90 programming environment.
- **Memory required to execute with typical data:** Negligible on the scale of typical applications calling the library.
- **Typical running time:** A small fraction (typically a few percent) of the running time of applications calling the library.
- **Purpose of program:** Provide simple and reproducible, yet realistic, parameterizations of the e^\pm - and γ -beam spectra for linear colliders.
- **Nature of physical problem:** The intricate beam dynamics in the interaction region of a high luminosity linear collider at $\sqrt{s} = 500\text{GeV}$ result in non-trivial energy spectra of the scattering electrons, positrons and photons. Physics simulations require simple and reproducible, yet realistic, parameterizations of these spectra.
- **Method of solution:** Parameterization, curve fitting, Monte Carlo event generation.
- **Keywords:** Event generation, beamstrahlung, linear colliders.

1 Introduction

Despite the enormous quantitative success of the electro-weak standard model up to energies of 200GeV, neither the nature of electro-weak symmetry breaking (EWSB) nor the origin of mass are understood.

From theoretical considerations, we know that clues to the answer of these open questions are hidden in the energy range below $\Lambda_{\text{EWSB}} = 4\pi v \approx 3.1\text{TeV}$. Either we will discover a Higgs particle in this energy range or signatures for a strongly interacting EWSB sector will be found. Experiments at CERN's Large Hadron Collider (LHC) will shed a first light on this regime in the next decade. In the past it has been very fruitful to complement experiments at high energy hadron colliders with experiments at e^+e^- -colliders. The simpler initial state allows more precise measurements with smaller theoretical errors. Lucid expositions of the physics opportunities of high energy e^+e^- colliders with references to the literature can be found in [1].

However, the power emitted by circular storage rings in form of synchrotron radiation scales like $(E/m)^4/R^2$ with the energy and mass of the particle and the radius of the ring. This cost becomes prohibitive after LEP2 and a Linear Collider (LC) has to be built instead.

Unfortunately, the “interesting” hard cross sections scale like $1/s$ with the square of the center of mass energy and a LC will have to operate at extremely high luminosities in excess of $10^{33}\text{cm}^{-2}\text{s}^{-1}$. To achieve such luminosities, the bunches of electrons and positrons have to be very dense. Under these conditions, the electrons undergo acceleration from strong electromagnetic forces from the positron bunch (and vice versa). The resulting synchrotron radiation is called *beamstrahlung* [2] and has a strong effect on the energy spectrum $D(x_1, x_2)$ of the colliding particles. This changes the observable e^+e^- cross sections

$$\frac{d\sigma_0^{e^+e^-}}{d\Omega}(s) \rightarrow \frac{d\sigma^{e^+e^-}}{d\Omega}(s) = \int_0^1 dx_1 dx_2 D_{e^+e^-}(x_1, x_2; \sqrt{s}) J(\Omega', \Omega) \frac{d\sigma_0^{e^+e^-}}{d\Omega'}(x_1 x_2 s) \quad (1a)$$

and produces luminosity for $e^\pm\gamma$ and $\gamma\gamma$ collisions:

$$\frac{d\sigma^{e^\pm\gamma}}{d\Omega}(s) = \int_0^1 dx_1 dx_2 D_{e^\pm\gamma}(x_1, x_2; \sqrt{s}) J(\Omega', \Omega) \frac{d\sigma_0^{e^\pm\gamma}}{d\Omega'}(x_1 x_2 s) \quad (1b)$$

$$\frac{d\sigma^{\gamma\gamma}}{d\Omega}(s) = \int_0^1 dx_1 dx_2 D_{\gamma\gamma}(x_1, x_2; \sqrt{s}) J(\Omega', \Omega) \frac{d\sigma_0^{\gamma\gamma}}{d\Omega'}(x_1 x_2 s) \quad (1c)$$

Therefore, simulations of the physics expected at a LC need to know the spectra of the e^\pm and γ beams precisely.

Microscopic simulations of the beam dynamics are available (e.g. ABEL[3], CAIN[4] and Guinea-Pig[5]) and their predictions are compatible with each other. But they require too much computer time and memory for direct use in physics programs. Circe1 provides a fast and simple parameterization of the results from these simulations. Furthermore, even if the computational cost of the simulations would be negligible, the input parameters for microscopic simulations are not convenient for particle physics applications. Due to the highly

	SBAND	TESLA	XBAND	SBAND	TESLA	XBAND
E/GeV	250	250	250	500	500	500
$N_{\text{particles}}/10^{10}$	1.1	3.63	0.65	2.9	1.8	0.95
$\epsilon_x/10^{-6}\text{mrad}$	5	14	5	10	14	5
$\epsilon_y/10^{-6}\text{mrad}$	0.25	0.25	0.08	0.1	0.06	0.1
β_x^*/mm	10.98	24.95	8.00	32	25	10.00
β_y^*/mm	0.45	0.70	0.13	0.8	0.7	0.12
σ_x/nm	335	845	286	571.87	598.08	226
σ_y/nm	15.1	18.9	4.52	9.04	6.55	3.57
$\sigma_z/\mu\text{m}$	300	700	100	500	500	125
f_{rep}	50	5	180	50	5	180
n_{bunch}	333	1135	90	125	2270	90

Table 1: Accelerator parameters for three typical designs at $\sqrt{s} = 500\text{GeV}$ and $\sqrt{s} = 1\text{TeV}$. The resulting distributions are shown in figure 1. The design efforts are currently concentrated on a 350GeV-800GeV LC. Therefore the Tesla parameters for 1TeV are slightly out of date.

non-linear beam dynamics, the optimization of LC designs is a subtle art [6], that is best practiced by the experts. Furthermore, particle physics applications need benchmarking and easily reproducible parameterizations are required for this purpose.

The parameterizations in **Circe1** are not based on approximate solutions (cf. [7]) of the beamstrahlung dynamics. Instead, they provide a “phenomenological” description of the results from full simulations. The parameterizations are as simple as possible while remaining consistent with basic physical principles:

1. *positivity*: the distribution functions $D(x_1, x_2)$ *must not* be negative in the physical region $[0, 1] \times [0, 1]$.
2. *integrability*: the definite integral of the distribution functions over the physical region $[0, 1] \times [0, 1]$ *must* exist, even though the distributions can have singularities.

This paper is organized as follows: I start in section 2 with a discussion of the input for the microscopic simulations. In section 3 I describe the usage of the **Circe1** library and in section 4 I discuss some technical details of the implementation. After discussing the parameterizations available (in internal version 2.2) in section 5, I conclude in section 8.

2 Parameters

The microscopic simulation program **Guinea-Pig** [5] used for the current version of the parameterizations in **Circe1** simulates the passage of electrons through a

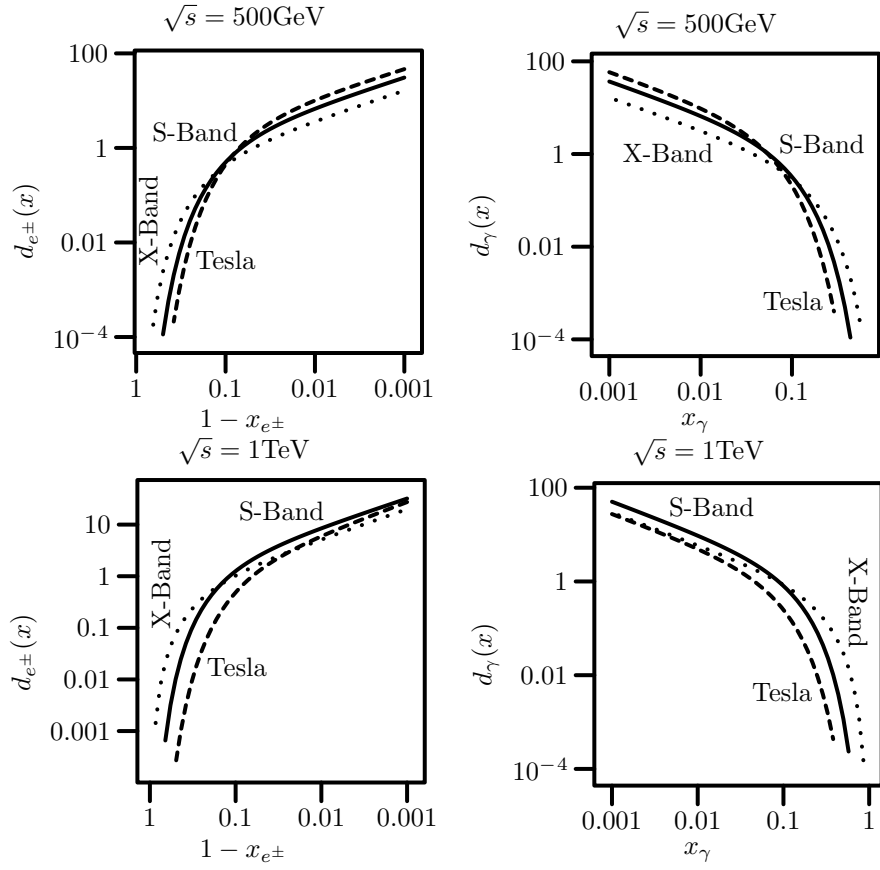


Figure 1: Version 1, revision 1996 09 02 of the factorized e^\pm - and γ -distributions at $\sqrt{s} = 500 \text{ GeV}$ and $\sqrt{s} = 1 \text{ TeV}$ in a doubly logarithmic plot. The accelerator parameters are taken from table 1.

	TESLA	TESLA	TESLA
E/GeV	175	250	400
$N_{\text{particles}}/10^{10}$	3.63	3.63	3.63
$\epsilon_x/10^{-6}\text{mrad}$	14	14	14
$\epsilon_y/10^{-6}\text{mrad}$	0.25	0.25	0.1
β_x^*/mm	25.00	24.95	15.00
β_y^*/mm	0.70	0.70	0.70
σ_x/nm	1010.94	845	668.67
σ_y/nm	22.6	18.9	9.46
$\sigma_z/\mu\text{m}$	700	700	700
f_{rep}	5	5	5
n_{bunch}	1135	1135	1135

Table 2: Accelerator parameters for the Tesla design at three planned [8] energies. The resulting distributions are shown in figure 2.

	High- \mathcal{L}	Low- \mathcal{L}	Low- ϵ_y
E/GeV	400	400	400
$N_{\text{particles}}/10^{10}$	3.63	3.63	1.800
$\epsilon_x/10^{-6}\text{mrad}$	14	14	12
$\epsilon_y/10^{-6}\text{mrad}$	0.1	0.25	0.025
β_x^*/mm	15.00	25.00	25.00
β_y^*/mm	0.70	0.70	0.50
σ_x/nm	668.67	700.00	
σ_y/nm	9.46		
$\sigma_z/\mu\text{m}$	700	700	500
f_{rep}	5	5	3
n_{bunch}	1135	1135	2260

Table 3: Variant accelerator parameters for the Tesla design at 800 GeV.

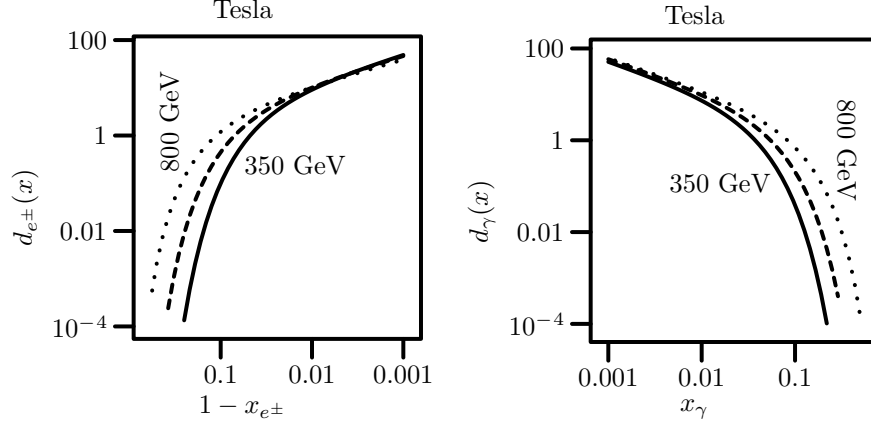


Figure 2: Version 1, revision 1996 09 02 of the factorized e^\pm - and γ -distributions for Tesla in a doubly logarithmic plot. The accelerator parameters are taken from table 2.

	TESLA	TESLA
E/GeV	250	400
$N_{\text{particles}}/10^{10}$	2	1.40
$\epsilon_x/10^{-6}\text{m rad}$	10	8
$\epsilon_y/10^{-6}\text{m rad}$	0.03	0.01
β_x^*/mm	15.00	15.00
β_y^*/mm	0.40	0.30
σ_x/nm	553	391
σ_y/nm	5	2
$\sigma_z/\mu\text{m}$	400	300
f_{rep}	5	3
n_{bunch}	2820	4500

Table 4: Accelerator parameters for a high luminosity Tesla design at two planned [8] energies. The resulting distributions are shown in figure 3.

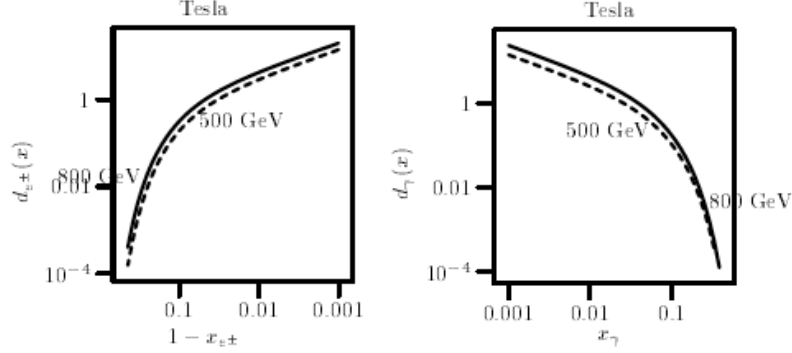


Figure 3: Version 5, revision 1998 05 05 of the factorized e^{\pm} - and γ -distributions for a high luminosity Tesla in a doubly logarithmic plot. The accelerator parameters are taken from table 4.

bunch of electrons (and vice versa). It takes the following accelerator parameters as input:

E : the energy of the particles before the beam-beam interaction.

$N_{\text{particles}}$: the number of particles per bunch.

$\epsilon_{x,y}$: the normalized horizontal and vertical emittances.

$\beta_{x,y}^*$: the horizontal and vertical beta functions.

$\sigma_{x,y,z}$: the horizontal, vertical and longitudinal beam size. A Gaussian shape is used for the charge distribution in the bunches.

f_{rep} : the repetition rate.

n_{bunch} : the number of bunches per train.

The transversal beam sizes, beta functions and normalized emittances for relativistic particles are related by

$$\beta_{x,y}^* = \frac{\sigma_{x,y}^2}{\epsilon_{x,y}} \frac{E}{m_e} \quad (2)$$

The parameters used in the most recent revision of the parameterizations are collected in tables 1 and 2. The resulting factorized electron/positron and photon distributions in version 1 of the parameterizations are depicted in figures 1 and 2.

The most important purpose of **Circe1** is to map the manifold of possible beam spectra for the NLC to a *finite* number of *reproducible* parameterizations. The distributions

$$D_{p_1 p_2}^{\alpha \nu \rho}(x_1, x_2; \sqrt{s}) \quad (3)$$

provided by **Circe1** are indexed by three integers

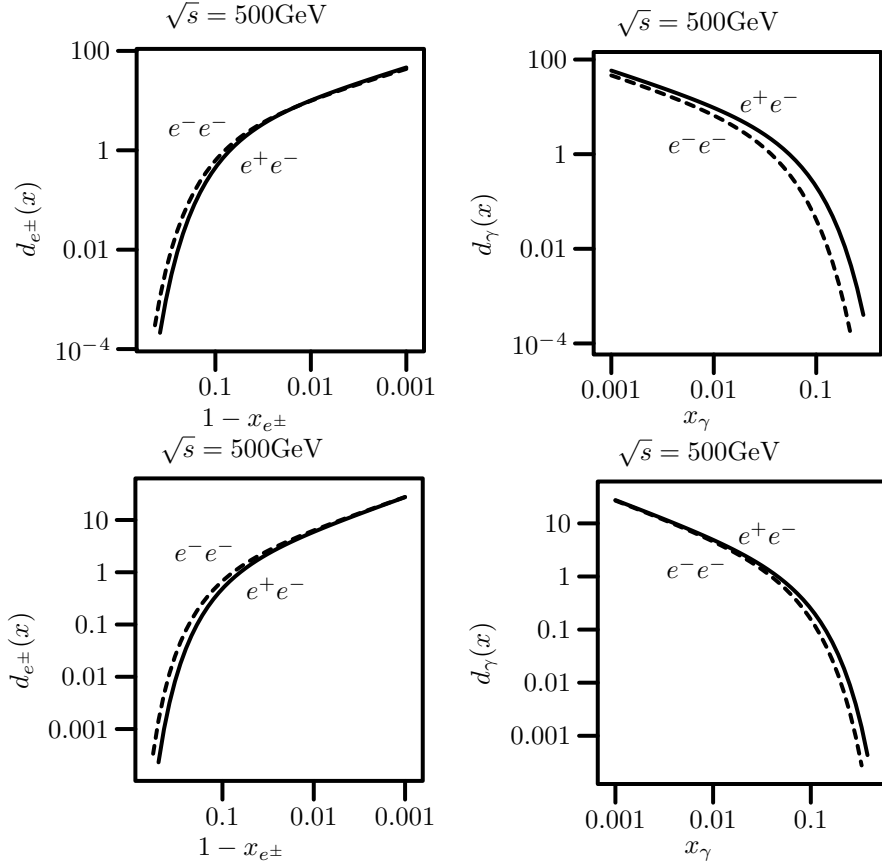


Figure 4: *Experimental:* Version 1, revision 0 of the factorized e^- - and γ -distributions for Tesla- e^-e^- in a doubly logarithmic plot. The accelerator parameters are taken from table 2 and have *not* been endorsed for use in an e^-e^- -machine yet!.

α : the *accelerator design class*: currently there are three options: S-band [9], Tesla [8], X-band [10, 11]. More variety will be added later, in particular the e^-e^- mode and the $e^-\gamma$ and $\gamma\gamma$ laser backscattering modes of these designs.

ν : the *version of the parameterization*: over the years, the form of the parameterizations can change, either because better approximations are found or because new simulation programs become available. All versions will remain available in order to be able to reproduce calculations.

ρ : the *revision date for the parameterization*: a particular parameterization can contain bugs, which will be fixed in subsequent revisions. While only the most recent revision should be used for new calculations, old revisions will remain available in order to be able to reproduce calculations.

The continuous parameter \sqrt{s} in (3) is misleading, because accelerator parameters have been optimized for discrete values of the energy. Therefore the distributions are not available for all values of \sqrt{s} .

The usage of the distributions in application programs is discussed in section 3.1. `Circe1` provides for each of the distributions a non-uniform random variate generator, that generates energy fractions according to the distributions. The usage of these generators is discussed in section 3.2.

3 Usage

3.1 Distributions

A generic interface to all distributions $D_{p_1 p_2}(x_1, x_2)$ is given by the `circe` function

11a \langle API documentation 11a $\rangle \equiv$

```
function circe, d, x1, x2
    real(kind=double) :: circe
    integer :: p1, p2
    d = circe (x1, x2, p1, p2)
```

Uses `circe` 31b.

where the energy fractions are specified by $x_{1,2}$ and the particles $p_{1,2}$ are identified by their standard Monte Carlo codes (we use C1 as a prefix to avoid name clashes when using CIRCE1 inside WHIZARD):[13]

11b \langle Particle codes 11b $\rangle \equiv$

```
integer, parameter, public :: C1_ELECTRON = 11
integer, parameter, public :: C1_POSITRON = -11
integer, parameter, public :: C1_PHOTON = 22
```

Defines:

C1_ELECTRON, used in chunks 21e, 31b, 73b, 80c, and 81a.
C1_PHOTON, used in chunks 31b, 73b, 80c, 81a, and 87b.
C1_POSITRON, used in chunks 22 and 81a.

The distributions can have integrable singularities at the end points, therefore the calling functions *must not* evaluate them at the endpoints 0 and 1. This is usually not a problem, since standard mapping techniques (cf. (10) below) will have to be used to take care of the singularity anyway. Nevertheless, all applications should favor open quadrature formulae (i.e. formulae not involving the endpoints) over closed formulae. The distributions are guaranteed to vanish unless $0 < x_{1,2} < 1$, with two exceptions. Firstly, the value -1 allows to pick up the integral of the continuum contribution:

$$D_{p_1 p_2}(-1, x_2) = \lim_{\epsilon \rightarrow +0} \int_{\epsilon}^{1-\epsilon} dx_1 D_{p_1 p_2}(x_1, x_2) \quad (4a)$$

$$D_{p_1 p_2}(x_1, -1) = \lim_{\epsilon \rightarrow +0} \int_{\epsilon}^{1-\epsilon} dx_2 D_{p_1 p_2}(x_1, x_2) \quad (4b)$$

$$D_{p_1 p_2}(-1, -1) = \lim_{\epsilon \rightarrow +0} \int_{\epsilon}^{1-\epsilon} dx_1 dx_2 D_{p_1 p_2}(x_1, x_2) \quad (4c)$$

The other exception is that the strength of δ -function contributions at the endpoint can be picked up from the value at this endpoint:

$$D_{e^+ e^-}(x_1, x_2) = D_{e^+ e^-}(1, 1) \delta(1 - x_1) \delta(1 - x_2) + \text{smooth and single } \delta \quad (5a)$$

$$D_{e^\pm \gamma}(x_1, x_2) = D_{e^\pm \gamma}(1, x_2) \delta(1 - x_1) + \text{smooth} \quad (5b)$$

$$D_{\gamma e^\pm}(x_1, x_2) = D_{\gamma e^\pm}(x_1, 1) \delta(1 - x_2) + \text{smooth} \quad (5c)$$

The use of these special values is demonstrated in an example in section 3.1.1 below.

The distributions are normalized such that

$$\lim_{\epsilon \rightarrow +0} \int_{-\epsilon}^{1+\epsilon} dx_1 dx_2 D_{e^+ e^-}(x_1, x_2) = 1. \quad (6)$$

and the nominal $e^+ e^-$ -luminosity of the currently active accelerator design can be retrieved from the database with the subroutine `circel`. The value is given in units of

$$\text{fb}^{-1} v^{-1} = 10^{32} \text{cm}^{-2} \text{sec}^{-1} \quad (7)$$

where $v = 10^7 \text{sec} \approx \text{year}/\pi$ is an “effective year” of running with about 30% up-time.

12a `<API documentation 11a>+≡`
`real(kind=dobule) :: lumi`
`call circel (lumi)`

Uses `circel 41f`.

A particular parameterization is selected by the `circes` function:

12b `<API documentation 11a>+≡`
`real(kind=double) :: x1m, x2m, roots`
`integer :: acc, ver, rev, chat`
`call circes (x1m, x2m, roots, acc, ver, rev, chat)`

Uses `circes 32b`.

The parameter **roots** corresponds to the nominal center of mass energy \sqrt{s} /GeV of the collider. Currently $\sqrt{s} = 350\text{GeV}, 500\text{GeV}, 800\text{GeV}, 1\text{TeV}$ (i.e. 350D0, 500D0, 800D0 and 1000D0) are supported. Application programs can *not* assume that energy values are interpolated. For convenience, e.g. in top threshold scans around 350GeV, a small interval around the supported values will be accepted as synonymous with the central value, but a warning will be printed. Section 5 should be consulted for the discrete values supported by a particular version of the parameterizations. Negative values of **roots** will keep the currently active value for \sqrt{s} .

The parameters **x1m** and **x2m** will set thresholds $x_{1,\min}$ and $x_{2,\min}$ for the event generation in the routines described in section 3.2.

The parameter **acc** selects the accelerator design. Currently the following accelerator codes are recognized:

13a $\langle \text{Accelerator codes 13a} \rangle \equiv$

```

integer, parameter :: SBAND = 1
integer, parameter :: TESLA = 2
integer, parameter :: XBAND = 3
integer, parameter :: JLCNLC = 3
integer, parameter :: SBNDEE = 4
integer, parameter :: TESLEE = 5
integer, parameter :: XBNDEE = 6
integer, parameter :: NLCH = 7
integer, parameter :: ILC = 8
integer, parameter :: CLIC = 9

```

Defines:

CLIC, used in chunk 35d.
 ILC, used in chunks 35d, 69b, 71, and 92.
 JLCNLC, used in chunks 17b, 18, 35d, 57b, 60–63, 66c, 67c, and 92.
 SBAND, used in chunks 35d, 40e, 41a, 44–47, 50, 92, 107c, and 108c.
 SBNDEE, used in chunks 34c, 35d, 41c, 45–47, 49, and 92.
 TESLA, used in chunks 32f, 33a, 35d, 40e, 41a, 44–47, 50–57, 59d, 60b, 62, 66b, 67b, 91b, 92, 107c, 108c, and 110a.
 TESLEE, used in chunks 34c, 35d, 49a, and 92.
 XBAND, used in chunks 40e, 41a, 44–47, 50, 57e, 107c, and 108c.
 XBNDEE, used in chunks 34c, 35d, 49, 91b, and 92.

The total number of accelerator codes

13b $\langle \text{Accelerator codes 13a} \rangle + \equiv$

```

integer, parameter :: NACC = 9

```

Defines:

NACC, used in chunks 17b, 34, 35, 40, 41, 45–47, 52a, 54a, 56c, 59c, 62a, and 66a.

The **ver** parameter is used to determine the version as follows:

ver > 0 : a frozen version which is documented in section 5. For example, version 1 is a family of factorized Beta distributions: $D(x_1, x_2) \propto x_1^{a_1} (1 - x_1)^{b_1} x_2^{a_2} (1 - x_2)^{b_2}$.

ver = 0 : the latest experimental version, which is usually not documented and can change at any time without announcement.

ver < 0 : keep the currently active version.

The **rev** parameter is used to determine the revision of a version as follows:

`rev > 0` : a frozen revision which is documented in section 5. The integer `rev` is constructed from the date as follows: `rev = 104 · year + 102 · month + day`, where the year is greater than 1995. Since Fortran77 ignored whitespace, it could be written like `1996 07 11` for readability. In Fortran90 the white space have been erased. If there is no exact match, the most recent revision before the specified date is chosen.

`rev = 0` : the most recent revision.

`rev < 0` : keep the currently active revision.

Finally, the parameter `chat` controls the “chattiness” of `circe`. If it is 0, only error messages are printed. If it is 1, the parameters in use are printed whenever they change. Higher values of `chat` can produce even more diagnostics.

In addition to the generic interface `circe`, there are specialized functions for particular particle distributions. Obviously

$$D_{e^\pm\gamma}^{\alpha\nu\rho}(x_1, x_2, s) = D_{\gamma e^\pm}^{\alpha\nu\rho}(x_2, x_1, s) \quad (8)$$

and there are three independent functions $D_{e^-e^+}$, $D_{e^-\gamma}$ and $D_{\gamma\gamma}$ for the e^+e^- colliders with reasonable mnemonics:

14 `<API documentation 11a>+≡`
`real(kind=double) :: circee, circeg, circgg`
`d = circee (x1, x2)`
`d = circeg (x1, x2)`
`d = circgg (x1, x2)`

Uses `circee` 41h, `circeg` 42c, and `circgg` 43c.

Calling the latter three functions is marginally faster in the current implementation, but this can change in the future.

3.1.1 Example

For clarification, let me give a simple example. Imagine we want to calculate the integrated production cross section

$$\sigma_X(s) = \int dx_1 dx_2 \sigma_{e^+e^- \rightarrow X}(x_1 x_2 s) D_{e^+e^-}(x_1, x_2, s) \quad (9)$$

Since the distributions are singular in the $x_{1,2} \rightarrow 1$ limit, we have to map away this singularity with

$$x \rightarrow t = (1 - x)^{1/\eta} \quad (10a)$$

Therefore

$$\int_0^1 dx f(x) = \int_0^1 dt \eta t^{\eta-1} f(1 - t^\eta) \quad (10b)$$

with η sufficiently large to give the integrand a finite limit at $x \rightarrow 1$. If f diverges like a power $f(x) \propto 1/(1 - x)^\beta$, this means $\eta > 1/(1 - \beta)$.

As a specific example, let us “measure” a one particle s -channel exchange cross section

$$\sigma(s) \propto \frac{1}{s} \quad (11)$$

```

15a <circe1_sample.f90: public 15a>≡
    public :: sigma
    Uses sigma 15b.

15b <circe1_sample.f90: subroutines 15b>≡
    function sigma (s)
    real(kind=double) :: s, sigma
    sigma = 1d0 / s
    end function sigma

```

Defines:
 sigma, used in chunks 15–18 and 20d.

I will present the example code in a bottom-up fashion, which should be intuitive and is described in some more detail in appendix A. Assuming the existence of a one- and a two-dimensional Gaussian integration function `gauss1` and `gauss2`,¹ we can perform the integral as follows:

```

15c <Gauss integration 15c>≡
    s = sigma (1d0) * circee (1d0, 1d0) &
    + gauss1 (d1, 0d0, 1d0, EPS) &
    + gauss1 (d2, 0d0, 1d0, EPS) &
    + gauss2 (d12, 0d0, 1d0, 0d0, 1d0, EPS)
    write (*, 1000) 'delta(sigma) (Gauss) =', (s-1d0)*100d0
    1000 format (1X, A22, 1X, F6.2, '%')

```

Uses `circee` 41h, `d1` 16a, `d12` 15e, `d2` 16c, `gauss1` 89f, `gauss2` 90d, and `sigma` 15b.

Note how the four combinations of continuum and δ -peak are integrated separately, where you have to use three auxiliary functions `d1`, `d2` and `d12`. The continuum contribution, including the Jacobian:

```

15d <circe1_sample.f90: public 15a>+≡
    public :: d12
    Uses d12 15e.

15e <circe1_sample.f90: subroutines 15b>+≡
    function d12 (t1, t2)
    real(kind=double) :: d12, t1, t2, x1, x2
    <EPS & PWR 16d>
    x1 = 1d0 - t1**PWR
    x2 = 1d0 - t2**PWR
    d12 = PWR*PWR * (t1*t2)**(PWR-1d0) &
    * sigma (x1*x2) * circee (x1, x2)
    end function d12

```

Defines:
 d12, used in chunk 15.
 Uses `circee` 41h and `sigma` 15b.

the first product of continuum and δ -peak:

```

15f <circe1_sample.f90: public 15a>+≡
    public :: d1
    Uses d1 16a.

```

¹They are provided in the example program `circe1_sample.f90`.

```

16a <circe1_sample.f90: subroutines 15b>+≡
      function d1 (t1)
        real(kind=double) :: t1, x1, d1
        <EPS & PWR 16d>
        x1 = 1d0 - t1**PWR
        d1 = PWR * t1**(PWR-1d0) * sigma (x1) * circee (x1, 1d0)
      end function d1

```

Defines:

d1, used in chunks 15, 16, 41–43, and 74–77.

Uses circee 41h and sigma 15b.

and the second one:

```

16b <circe1_sample.f90: public 15a>+≡
      public :: d2

```

Uses d2 16c.

```

16c <circe1_sample.f90: subroutines 15b>+≡
      function d2 (t2)
        real(kind=double) :: t2, x2, d2
        <EPS & PWR 16d>
        x2 = 1d0 - t2**PWR
        d2 = PWR * t2**(PWR-1d0) * sigma (x2) * circee (1d0, x2)
      end function d2

```

Defines:

d2, used in chunks 15, 16, 41–43, and 74–77.

Uses circee 41h and sigma 15b.

Below you will see that the power of the singularity of the e^+e^- distributions at $x \rightarrow 1$ is $\approx -2/3$. To be on the safe side, we choose the power η in (10) as 5. It is kept in the parameter PWR, while EPS is the desired accuracy of the Gaussian integration:

```

16d <EPS & PWR 16d>≡
      real(kind=double), parameter :: EPS = 1d-6, PWR = 5d0

```

The Gauss integration of the non-singular version converges to the cotrrect value only if the final bin is integrated separately:

```

16e <Second Gauss integration 16e>≡
      s = gauss2 (d12a, 0d0, 1d0-KIREPS, 0d0, 1d0-KIREPS, EPS) &
        + gauss2 (d12a, 0d0, 1d0-KIREPS, 1d0-KIREPS, 1d0, EPS) &
        + gauss2 (d12a, 1d0-KIREPS, 1d0, 0d0, 1d0-KIREPS, EPS) &
        + gauss2 (d12a, 1d0-KIREPS, 1d0, 1d0-KIREPS, 1d0, EPS)
      write (*, 1000) 'delta(sigma) (Gauss) =', (s-1d0)*100d0

```

Uses d12a 17a, gauss2 90d, and sigma 15b.

```

16f <EPS & PWR 16d>+≡
      real(kind=double), parameter :: KIREPS = 1D-6

```

```

16g <circe1_sample.f90: public 15a>+≡
      public :: d12a

```

Uses d12a 17a.

```

17a <circe1_sample.f90: subroutines 15b>+=
      function d12a (x1, x2)
        real(kind=double) :: x1, x2, d12a
        d12a = sigma (x1*x2) * kirkee (x1, x2)
      end function d12a

```

Defines:

d12a, used in chunk 16.

Uses kirkee 74b and sigma 15b.

These code fragments can now be used in a main program that loops over energies and accelerator designs

```

17b <circe1_sample.f90 17b>≡
      ! circe1_sample.f90 -- canonical beam spectra for linear collider physics
      ! $Id: prelude.nw 5455 2014-03-12 14:26:30Z ohl $
      <Copyleft notice 30a>
      module sample_routines
        use kinds
        use circe1 !NODEP!

        implicit none
        private

        <circe1_sample.f90: public 15a>

        contains

        <circe1_sample.f90: subroutines 15b>

      end module sample_routines

      program circe1_sample
        use kinds
        use sample_routines
        use circe1

        implicit none

        <Accelerator codes 13a>
        <EPS & PWR 16d>
        <Other variables in sample 19>
        integer :: acc, ver, i
        real(kind=double), dimension(9) :: roots(9) = &
          (/ 90D0, 170D0, 250D0, 350D0, 500D0, &
            800D0, 1000D0, 1200D0, 1500D0 /)
        do acc = 1, NACC
          ! do acc = JLCNLC, NLCH, NLCH-JLCNLC
          do ver = 9, 9
            do i = 1, 9
              call circes (0d0, 0d0, roots(i), acc, ver, 20020328, 1)
              <Gauss integration 15c>
            end do
          end do
        end do
      end program circe1_sample

```

```

                                <Second Gauss integration 16e>
                                <Monte Carlo integration 20d>
                        end do
                end do
        end do
end program circe1_sample

```

Uses JLCNLC 13a, NACC 13b, and circes 32b.

with the following result

```

18 <Sample output 18>≡
    circe1:message: starting up ...
    circe1:message: $Id: prelude.nw 5455 2014-03-12 14:26:30Z ohl $
    circe1:message: updating 'roots' to 90.0
    circe1:message: updating 'ver' to 7
    circe1:message: updating 'rev' to 20000501
    delta(sigma) (Gauss) = 0.11%
    delta(sigma) (MC)    = 0.11%
                        +/- 0.00%
    circe1:message: updating 'roots' to 170.0
    circe1:message: updating 'ver' to 7
    delta(sigma) (Gauss) = 0.38%
    delta(sigma) (MC)    = 0.38%
                        +/- 0.01%
    circe1:message: updating 'roots' to 350.0
    circe1:message: updating 'ver' to 7
    delta(sigma) (Gauss) = 1.67%
    delta(sigma) (MC)    = 1.66%
                        +/- 0.03%
    circe1:message: updating 'roots' to 500.0
    circe1:message: updating 'ver' to 7
    delta(sigma) (Gauss) = 3.66%
    delta(sigma) (MC)    = 3.58%
                        +/- 0.07%
    circe1:message: updating 'roots' to 800.0
    circe1:message: updating 'ver' to 7
    delta(sigma) (Gauss) = 5.21%
    delta(sigma) (MC)    = 5.19%
                        +/- 0.11%
    circe1:message: updating 'roots' to 1000.0
    circe1:message: updating 'ver' to 7
    circe1:message: energy 1000.0GeV too high, using spectrum for 800.0GeV
    delta(sigma) (Gauss) = 5.21%
    delta(sigma) (MC)    = 5.19%
                        +/- 0.11%
    circe1:message: updating 'roots' to 90.0
    circe1:message: updating 'acc' to JLCNLC
    circe1:message: updating 'ver' to 7
    circe1:message: energy 90.0GeV too low, using spectrum for 500.0GeV
    delta(sigma) (Gauss) = 4.74%

```

```

delta(sigma) (MC)    = 4.75%
                    +/- 0.11%
circe1:message: updating 'roots' to 170.0
circe1:message: updating 'ver' to 7
circe1:message: energy 170.0GeV too low, using spectrum for 500.0GeV
delta(sigma) (Gauss) = 4.74%
delta(sigma) (MC)    = 4.68%
                    +/- 0.11%
circe1:message: updating 'roots' to 350.0
circe1:message: updating 'ver' to 7
circe1:message: energy 350.0GeV too low, using spectrum for 500.0GeV
delta(sigma) (Gauss) = 4.74%
delta(sigma) (MC)    = 4.75%
                    +/- 0.11%
circe1:message: updating 'roots' to 500.0
circe1:message: updating 'ver' to 7
delta(sigma) (Gauss) = 4.74%
delta(sigma) (MC)    = 4.75%
                    +/- 0.11%
circe1:message: updating 'roots' to 800.0
circe1:message: updating 'ver' to 7
circe1:message: energy 800.0GeV interpolated between 500.0 and 1000.0GeV
delta(sigma) (Gauss) = 8.37%
delta(sigma) (MC)    = 8.39%
                    +/- 0.21%
circe1:message: updating 'roots' to 1000.0
circe1:message: updating 'ver' to 7
delta(sigma) (Gauss) = 15.39%
delta(sigma) (MC)    = 14.68%
                    +/- 0.33%

```

Uses JLCNLC 13a and sigma 15b.

We almost forgot to declare the variables in the main program

```

19 <Other variables in sample 19>≡
    real(kind=double) :: s

```

This concludes the integration example. It should have made it obvious how to proceed in a realistic application.

In section 3.2.1 below, I will describe a Monte Carlo method for calculating such integrals efficiently.

3.2 Generators

The function `circe` and its companions are opaque to the user. Since they will in general contain singularities, applications will *not* be able to generate corresponding samples of random numbers efficiently. To fill this gap, four random number generators are provided. The subroutine `girce` will generate particle types $p_{1,2}$ and energy fractions $x_{1,2}$ in one step, according to the selected distribution.² Particle p_1 will be either a positron or a photon and p_2 will be

²The implementation of the flavor selection with non-vanishing thresholds $x_{1,\min}$ and $x_{2,\min}$ is moderately inefficient at the moment. It can be improved by a factor of two.

either an electron or a photon. The energy fractions are guaranteed to be above the currently active thresholds: $x_i \geq x_{i,\min}$. This can be used to cut on soft events—the photon distributions are rather soft—which might not be interesting in most simulations.

20a *<API documentation 11a>+≡*
`call girce (x1, x2, p1, p2, rng)`

Uses `girce 80c`.

The output parameters of `girce` are identical to the input parameters of `circe`, with the exception of `rng`. The latter is a subroutine with a single double precision argument, which will be assigned a uniform deviate from the interval $[0, 1]$ after each call:

20b *<API documentation 11a>+≡*
`subroutine rng (r)`
`real(kind=double) :: r`
`r = <uniform deviate on [0, 1] (never defined)>`
`end subroutine rng`

Typically, it will be just a wrapper around the standard random number generator of the application program. For studies with a definite initial state, three generator functions are available.

20c *<API documentation 11a>+≡*
`call gircee (x1, x2, rng)`
`call girceg (x1, x2, rng)`
`call gircgg (x1, x2, rng)`

Uses `gircee 81e`, `girceg 82c`, and `gircgg 83c`.

3.2.1 Example

Returning to the example from section 3.2.1, I present a concise Monte Carlo algorithm for calculating the same integral:

20d *<Monte Carlo integration 20d>≡*
`s = 0d0`
`s2 = 0d0`
`do n = 1, NEVENT`
`call gircee (x1, x2, random)`
`w = sigma (x1*x2)`
`s = s + w`
`s2 = s2 + w*w`
`end do`
`s = s / dble(NEVENT)`
`s2 = s2 / dble(NEVENT)`
`write (*, 1000) 'delta(sigma) (MC) = ', (s-1d0)*100d0`
`write (*, 1000) ' +/- ', sqrt((s2-s*s)/dble(NEVENT))*100d0`

Uses `gircee 81e`, `random 21b`, and `sigma 15b`.

20e *<Other variables in sample 19>+≡*
`real(kind=double) :: w, s2, x1, x2`
`integer, parameter :: NEVENT = 10000`
`integer :: n`

Here is a simple linear congruential random number generator for the sample program. Real applications will use their more sophisticated generators instead.

```

21a <circe1_sample.f90: public 15a>+≡
    public :: random
    Uses random 21b.
21b <circe1_sample.f90: subroutines 15b>+≡
    subroutine random (r)
        real(kind=double), intent(out) :: r
        integer :: m = 259200, a = 7141, c = 54773
        integer, save :: n = 0
        ! data n /0/
        n = mod(n*a+c,m)
        r = real (n, kind=double) / real (m, kind=double)
    end subroutine random

```

Defines:

`random`, used in chunks 20 and 21.

If the cross section is slowly varying on the range where the $x_{1,2}$ distributions are non-zero, this algorithm is very efficient.

However, if this condition is not met, the explicit form of the parameterizations in section 5 should be consulted and appropriate mapping techniques should be applied. The typical example for this problem is a narrow resonance just below the nominal beam energy.

3.2.2 Event Generators

For Monte Carlo event generators that use the standard `/hepevt/` common block [14], the addition of the `Circe1` library is trivial. During the initialization of the event generator, the `circes` subroutine is called to set up `Circe1`'s internal state. For example:

```

21c <Initialize event generator 21c>≡
    call circes (0d0, 0d0, roots, acc, ver, 1996 07 11, 1)
    Uses circes 32b.

```

During event generation, before setting up the e^+e^- initial state, the `gircee` subroutine is called with the event generator's random number generator:

```

21d <Event generation 21d>≡
    call gircee (x1, x2, random)
    Uses gircee 81e and random 21b.

```

The resulting energy fractions x_1 and x_2 are now available for defining the initial state electron

```

21e <Event generation 21d>+≡
    isthep(1) = 101
    idhep(1) = C1_ELECTRON
    phep(1,1) = 0d0
    phep(2,1) = 0d0
    phep(3,1) = x1 * ebeam
    phep(4,1) = x1 * ebeam
    phep(5,1) = 0d0
    Uses C1_ELECTRON 11b.

```

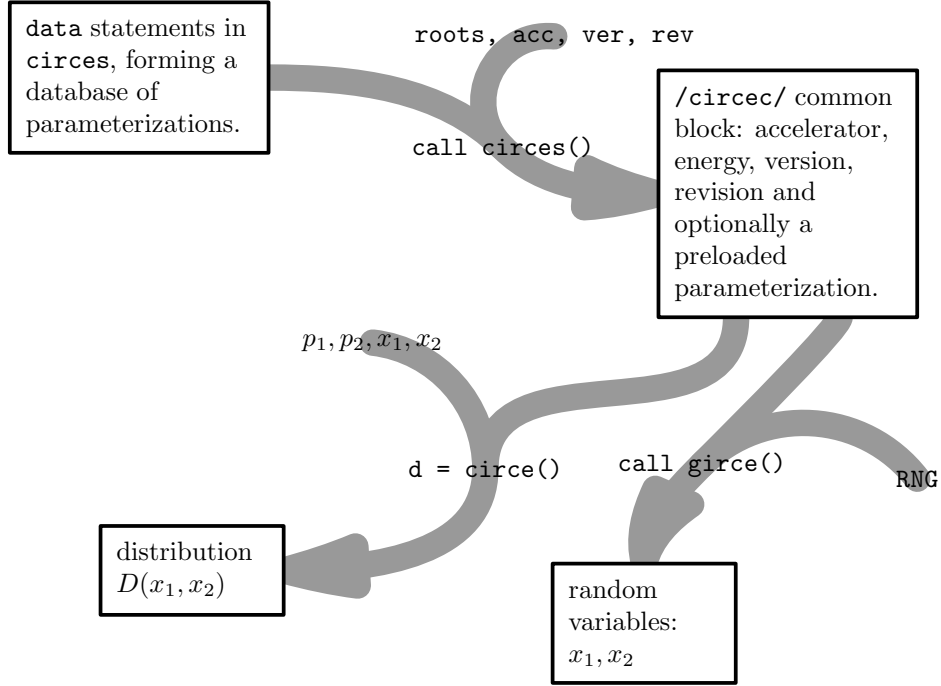


Figure 5: Architecture of **Circe1**: `circes()` selects energy and accelerator and loads the parameterization. The function `circe()` calculates the values of the selected distribution function at the given energy fractions. The subroutine `girce()` generates energy fractions using a specified random number generator in accordance with the selected distribution.

and positron.

```

22 (Event generation 21d)+≡
    isthep(2) = 102
    idhep(2) = C1_POSITRON
    phep(1,2) = 0d0
    phep(2,2) = 0d0
    phep(3,2) = - x2 * ebeam
    phep(4,2) = x2 * ebeam
    phep(5,2) = 0d0

```

Uses C1_POSITRON 11b.

Using **Circe1** with other event generators should be straightforward as well.

4 Technical Notes

The structure of **Circe1** is extremely simple (cf. figure 5) and is mainly a book-keeping exercise. All that needs to be done is to maintain a database of available parameterizations and to evaluate the corresponding functions. The only non trivial algorithms are used for the efficient generation of random deviates.

	SBAND	TESLA	TESLA'	XBAND
$\mathcal{L}/\text{fb}^{-1}\nu^{-1}$	$31.38^{+0.22}_{-0.22}$	$106.25^{+0.71}_{-0.71}$	$95.24^{+0.73}_{-0.73}$	$36.39^{+0.29}_{-0.29}$
$\int d_{e\pm}$	$0.4812^{+0.0041}_{-0.0041}$	$0.5723^{+0.0046}_{-0.0045}$	$0.3512^{+0.0048}_{-0.0048}$	$0.3487^{+0.0040}_{-0.0040}$
$x_{e\pm}^\alpha$	$11.1534^{+0.0770}_{-0.0761}$	$15.2837^{+0.0923}_{-0.0914}$	$27.1032^{+0.3071}_{-0.3019}$	$6.9853^{+0.0733}_{-0.0718}$
$(1 - x_{e\pm})^\alpha$	$-0.6302^{+0.0013}_{-0.0012}$	$-0.6166^{+0.0011}_{-0.0011}$	$-0.6453^{+0.0017}_{-0.0017}$	$-0.6444^{+0.0017}_{-0.0017}$
$\int d_\gamma$	$0.6237^{+0.0033}_{-0.0033}$	$0.7381^{+0.0036}_{-0.0036}$	$0.3502^{+0.0034}_{-0.0034}$	$0.4149^{+0.0031}_{-0.0031}$
x_γ^α	$-0.6911^{+0.0006}_{-0.0006}$	$-0.6921^{+0.0006}_{-0.0006}$	$-0.6947^{+0.0011}_{-0.0011}$	$-0.6876^{+0.0010}_{-0.0010}$
$(1 - x_\gamma)^\alpha$	$14.9355^{+0.0761}_{-0.0754}$	$24.1647^{+0.1124}_{-0.1116}$	$33.6576^{+0.3021}_{-0.2983}$	$8.3227^{+0.0659}_{-0.0649}$

Table 5: Version 1, revision 1997 04 16 of the beam spectra at 500 GeV. The rows correspond to the luminosity per effective year, the integral over the continuum and the powers in the factorized Beta distributions (12).

I have avoided the use of initialized `common` blocks (i.e. `block data` subroutines), because the Fortran77 standard does not provide a *portable* way of ensuring that `block data` subroutines are actually executed at loading time³. Instead, the `/circom/` common block is tagged by a “magic number” to check for initialization and its members are filled by the `circes` subroutine when necessary.

A more flexible method would be to replace the `data` statements by reading external files. This option causes portability problems, however, because I would have to make sure that the names of the external files are valid in all files systems of the target operating systems. More significantly, splitting the implementation into several parts forces the user to keep all files up to date. This can be a problem, because Fortran source files and data input files will typically be kept in different parts of the file system.

The option of implementing `Circe1` statelessly, i.e. with pure function calls and without `common` blocks, has been dismissed. While it would have been more straightforward on the side of the library, it would have placed the burden of maintaining state (accelerator, energy, etc.) on the application program, thereby complicating them considerably. Keeping an explicit state in `Circe1` has the additional benefit of allowing to precompute certain internal variables, resulting in a more efficient implementation.

5 Parameterizations

The internal Version 2.2 of `Circe11` supports just one version of the parameterizations. Future versions will provide additional parameterizations.

5.1 Version 1

The first version of the parameterization uses a simple factorized *ansatz*

$$D_{p_1 p_2}^{\alpha 1 \rho}(x_1, x_2, s) = d_{p_1}^{\alpha 1 \rho}(x_1) d_{p_2}^{\alpha 1 \rho}(x_2) \quad (12a)$$

³In Fortran90 the common blocks have been replaced by saved module variables.

	SBAND	TESLA	TESLA'	XBAND
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$119.00^{+0.83}_{-0.83}$	214.33^{+0***}_{-0***}	212.22^{+0***}_{-0***}	$118.99^{+0.91}_{-0.91}$
$\int d_{e\pm}$	$0.5604^{+0.0040}_{-0.0039}$	$0.6686^{+0.0040}_{-0.0040}$	$0.4448^{+0.0043}_{-0.0043}$	$0.5001^{+0.0038}_{-0.0038}$
$x_{e\pm}^\alpha$	$4.2170^{+0.0258}_{-0.0255}$	$5.5438^{+0.0241}_{-0.0239}$	$9.6341^{+0.0814}_{-0.0803}$	$2.6184^{+0.0192}_{-0.0190}$
$(1 - x_{e\pm})^\alpha$	$-0.6118^{+0.0013}_{-0.0013}$	$-0.5847^{+0.0011}_{-0.0011}$	$-0.6359^{+0.0014}_{-0.0014}$	$-0.6158^{+0.0015}_{-0.0015}$
$\int d_\gamma$	$0.7455^{+0.0032}_{-0.0032}$	$1.0112^{+0.0033}_{-0.0033}$	$0.4771^{+0.0031}_{-0.0031}$	$0.6741^{+0.0031}_{-0.0031}$
x_γ^α	$-0.6870^{+0.0006}_{-0.0006}$	$-0.6908^{+0.0004}_{-0.0004}$	$-0.6936^{+0.0008}_{-0.0008}$	$-0.6834^{+0.0007}_{-0.0007}$
$(1 - x_\gamma)^\alpha$	$6.7145^{+0.0310}_{-0.0308}$	$9.9992^{+0.0342}_{-0.0340}$	$13.1607^{+0.0896}_{-0.0886}$	$3.8589^{+0.0215}_{-0.0213}$

Table 6: Version 1, revision 1997 04 17 of the beam spectra at 1 TeV.

	350 GeV	500 GeV	800 GeV	1600 GeV
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$97.45^{+0.67}_{-0.67}$	$106.25^{+0.71}_{-0.71}$	170.86^{+0***}_{-0***}	340.86^{+0***}_{-0***}
$\int d_{e\pm}$	$0.6093^{+0.0049}_{-0.0049}$	$0.5723^{+0.0046}_{-0.0045}$	$0.6398^{+0.0042}_{-0.0041}$	$0.5094^{+0.0040}_{-0.0040}$
$x_{e\pm}^\alpha$	$17.6137^{+0.1065}_{-0.1055}$	$15.2837^{+0.0923}_{-0.0914}$	$7.6221^{+0.0365}_{-0.0361}$	$5.0550^{+0.0353}_{-0.0349}$
$(1 - x_{e\pm})^\alpha$	$-0.6061^{+0.0011}_{-0.0011}$	$-0.6166^{+0.0011}_{-0.0011}$	$-0.5944^{+0.0011}_{-0.0011}$	$-0.6187^{+0.0013}_{-0.0013}$
$\int d_\gamma$	$0.7729^{+0.0039}_{-0.0039}$	$0.7381^{+0.0036}_{-0.0036}$	$0.9178^{+0.0034}_{-0.0034}$	$0.5875^{+0.0031}_{-0.0031}$
x_γ^α	$-0.6949^{+0.0006}_{-0.0006}$	$-0.6921^{+0.0006}_{-0.0006}$	$-0.6908^{+0.0005}_{-0.0005}$	$-0.6892^{+0.0007}_{-0.0007}$
$(1 - x_\gamma)^\alpha$	$28.9399^{+0.1370}_{-0.1361}$	$24.1647^{+0.1124}_{-0.1116}$	$13.1167^{+0.0497}_{-0.0495}$	$7.5514^{+0.0428}_{-0.0424}$

Table 7: Version 1, revision 1997 04 17 of the beam spectra for TESLA.

	500 GeV	800 GeV
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$339.80^{+0.83}_{-0.83}$	$359.36^{+0.93}_{-0.93}$
$\int d_{e\pm}$	$0.5019^{+0.0016}_{-0.0016}$	$0.4125^{+0.0016}_{-0.0016}$
$x_{e\pm}^\alpha$	$12.2867^{+0.0318}_{-0.0316}$	$13.3242^{+0.0442}_{-0.0440}$
$(1 - x_{e\pm})^\alpha$	$-0.6276^{+0.0005}_{-0.0005}$	$-0.6401^{+0.0005}_{-0.0005}$
$\int d_\gamma$	$0.5114^{+0.0012}_{-0.0012}$	$0.3708^{+0.0011}_{-0.0011}$
x_γ^α	$-0.6912^{+0.0003}_{-0.0003}$	$-0.6924^{+0.0004}_{-0.0004}$
$(1 - x_\gamma)^\alpha$	$17.0673^{+0.0375}_{-0.0375}$	$16.8145^{+0.0482}_{-0.0480}$

Table 8: Version 5, revision 1998 05 05 of the beam spectra for high luminosity TESLA.

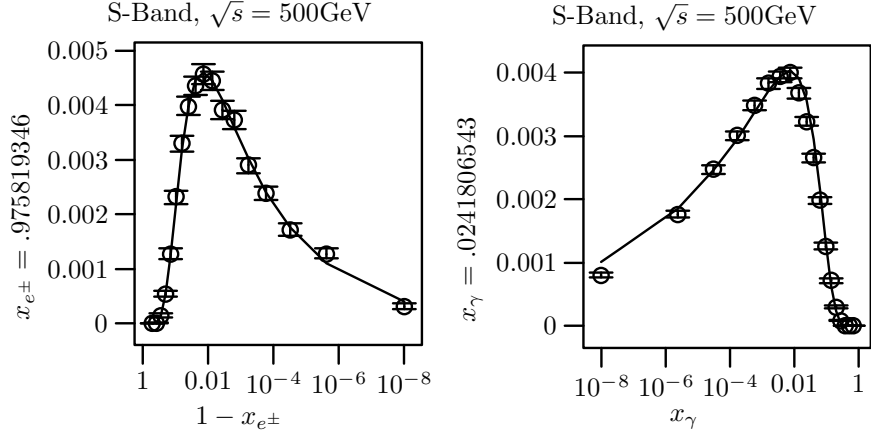


Figure 6: Fit of the e^\pm - and γ -distributions for the S-Band design at $\sqrt{s} = 500\text{GeV}$. The open circles with error bars are the result of the Guinea-Pig simulation. The full line is the fit.

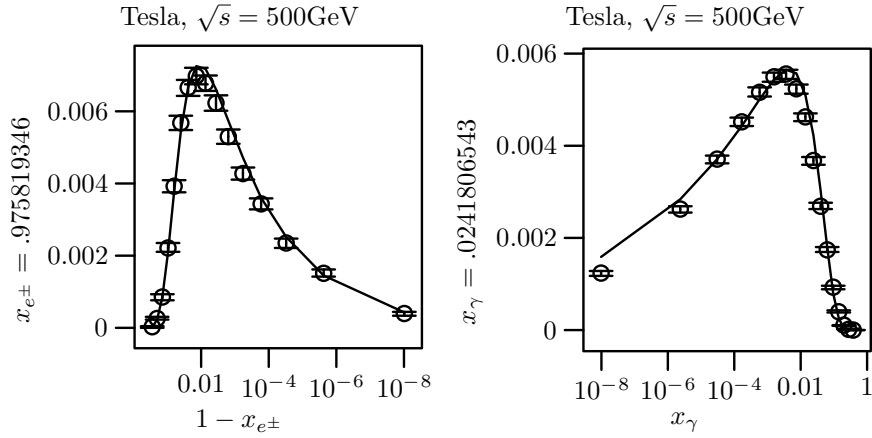


Figure 7: Fit of the e^\pm - and γ -distributions for the Tesla design at $\sqrt{s} = 500\text{GeV}$.

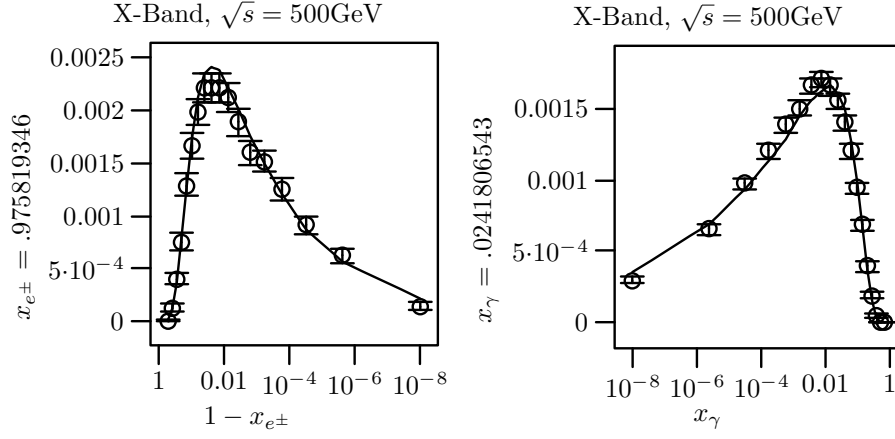


Figure 8: Fit of the e^\pm - and γ -distributions for the X-Band design at $\sqrt{s} = 500 \text{ GeV}$.

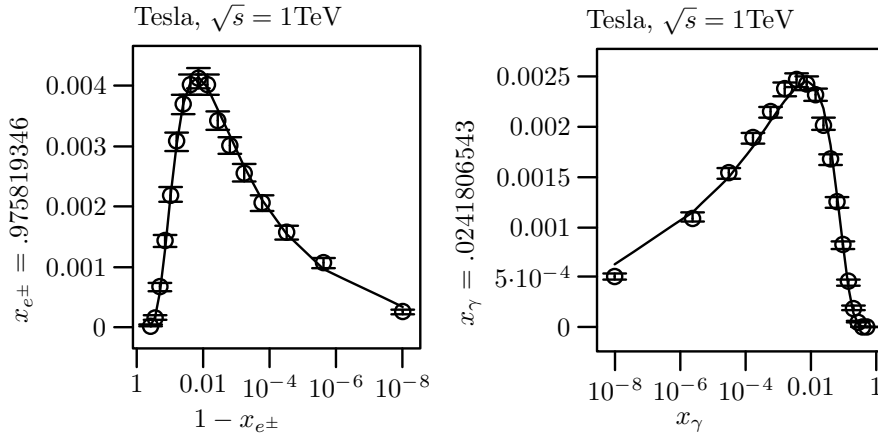


Figure 9: Fit of the e^\pm - and γ -distributions for the Tesla design at $\sqrt{s} = 1 \text{ TeV}$.

	SBNDEE	TESLEE	XBNDEE
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$9.29^{+0.06}_{-0.06}$	$21.62^{+0.17}_{-0.17}$	$13.97^{+0.10}_{-0.10}$
$\int d_{e^\pm}$	$.6513^{+0.0059}_{-0.0059}$	$.7282^{+0.0083}_{-0.0082}$	$.5270^{+0.0049}_{-0.0049}$
$x_{e^\pm}^\alpha$	$10.3040^{+0.0601}_{-0.0593}$	$14.8578^{+0.1047}_{-0.1034}$	$5.8897^{+0.0455}_{-0.0448}$
$(1 - x_{e^\pm})^\alpha$	$-.5946^{+0.0015}_{-0.0015}$	$-.5842^{+0.0018}_{-0.0018}$	$-.6169^{+0.0016}_{-0.0015}$
$\int d_\gamma$	$.4727^{+0.0035}_{-0.0035}$	$.5300^{+0.0046}_{-0.0046}$	$.3746^{+0.0029}_{-0.0029}$
x_γ^α	$-.6974^{+0.0009}_{-0.0009}$	$-.7039^{+0.0009}_{-0.0009}$	$-.6892^{+0.0010}_{-0.0010}$
$(1 - x_\gamma)^\alpha$	$20.6447^{+0.1513}_{-0.1497}$	$36.1286^{+0.3027}_{-0.2991}$	$10.0872^{+0.0822}_{-0.0815}$

Table 9: *Experimental* Version 1, revision 0 of the beam spectra at 500 GeV. The rows correspond to the luminosity per effective year, the integral over the continuum and the powers in the factorized Beta distributions (12).

	SBNDEE	TESLEE	XBNDEE
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$45.59^{+0.34}_{-0.34}$	$25.47^{+0.20}_{-0.20}$	$41.06^{+0.28}_{-0.28}$
$\int d_{e^\pm}$	$.7892^{+0.0075}_{-0.0074}$	$.6271^{+0.0066}_{-0.0065}$	$.7203^{+0.0058}_{-0.0058}$
$x_{e^\pm}^\alpha$	$5.4407^{+0.0285}_{-0.0281}$	$8.7504^{+0.0669}_{-0.0658}$	$2.7415^{+0.0121}_{-0.0119}$
$(1 - x_{e^\pm})^\alpha$	$-.5285^{+0.0020}_{-0.0020}$	$-.6058^{+0.0017}_{-0.0017}$	$-.5049^{+0.0020}_{-0.0020}$
$\int d_\gamma$	$.6403^{+0.0040}_{-0.0040}$	$.4278^{+0.0038}_{-0.0038}$	$.6222^{+0.0032}_{-0.0032}$
x_γ^α	$-.6960^{+0.0008}_{-0.0008}$	$-.6982^{+0.0010}_{-0.0010}$	$-.6795^{+0.0008}_{-0.0008}$
$(1 - x_\gamma)^\alpha$	$12.4803^{+0.0839}_{-0.0831}$	$18.5260^{+0.1674}_{-0.1655}$	$4.7506^{+0.0262}_{-0.0260}$

Table 10: *Experimental* Version 1, revision 0 of the beam spectra at 1 TeV.

where the distributions are simple Beta distributions:

$$d_{e^\pm}^{\alpha 1\rho}(x) = a_0^{\alpha\rho} \delta(1-x) + a_1^{\alpha\rho} x^{a_2^{\alpha\rho}} (1-x)^{a_3^{\alpha\rho}} \quad (12b)$$

$$d_\gamma^{\alpha 1\rho}(x) = a_4^{\alpha\rho} x^{a_5^{\alpha\rho}} (1-x)^{a_6^{\alpha\rho}} \quad (12c)$$

This form of the distributions is motivated by the observation [2] that the e^\pm distributions diverge like a power for $x \rightarrow 1$ and vanish at $x \rightarrow 0$. The behavior of the γ distributions is similar with the borders exchanged.

5.1.1 Fitting

The parameters a_i in (12) have been obtained by a least-square fit of (12) to histograms of simulation results from **Guinea-Pig**. Some care has to be taken when fitting singular distributions to histogrammed data. Obviously equidistant bins are not a good idea, because most bins will be almost empty (cf. figures 1 and 2) and consequently a lot of information will be wasted. One solution to this problem is the use of logarithmic bins. This, however, maps the compact region $[0, 1] \times [0, 1]$ to $[-\infty, 0] \times [-\infty, 0]$, which is inconvenient because of the missing lower bounds.

	350 GeV	500 GeV	800 GeV
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$15.18^{+0.13}_{-0.13}$	$21.62^{+0.17}_{-0.17}$	$43.98^{+0.38}_{-0.38}$
$\int d_{e\pm}$	$.6691^{+0.0083}_{-0.0083}$	$.7282^{+0.0083}_{-0.0082}$	$.7701^{+0.0090}_{-0.0089}$
$x_{e\pm}^\alpha$	$25.2753^{+0.2040}_{-0.2007}$	$14.8578^{+0.1047}_{-0.1034}$	$8.1905^{+0.0543}_{-0.0535}$
$(1 - x_{e\pm})^\alpha$	$-.5994^{+0.0017}_{-0.0017}$	$-.5842^{+0.0018}_{-0.0018}$	$-.5575^{+0.0021}_{-0.0021}$
$\int d_\gamma$	$.4464^{+0.0047}_{-0.0047}$	$.5300^{+0.0046}_{-0.0046}$	$.5839^{+0.0047}_{-0.0047}$
x_γ^α	$-.7040^{+0.0011}_{-0.0011}$	$-.7039^{+0.0009}_{-0.0009}$	$-.7046^{+0.0009}_{-0.0009}$
$(1 - x_\gamma)^\alpha$	$60.1882^{+0.5882}_{-0.5797}$	$36.1286^{+0.3027}_{-0.2991}$	$19.3944^{+0.1681}_{-0.1660}$

Table 11: *Experimental* Version 1, revision 0 of the beam spectra for TESLEE.

The more appropriate solution is to use two maps

$$\begin{aligned}\phi : [0, 1] &\rightarrow [0, 1] \\ x &\mapsto y = x^{1/\eta}\end{aligned}\tag{13}$$

where $x = x_\gamma$ or $x = 1 - x_{e\pm}$, and to bin the result equidistantly. If η is chosen properly (cf. (10)), the bin contents will then fall off at the singularity. The fits in tables 5, 6, and 7 have been performed with $\eta = 5$ and the resulting bin contents can be read off from figures 6–9.

Using this procedure for binning the results of the simulations, the popular fitting package MINUIT [15] converges quickly in all cases considered. The resulting parameters are given in tables 5, 6, and 7. Plots of the corresponding distributions have been shown in figures 1 and 2. It is obvious that an *ansatz* like (12) is able to distinguish among the accelerator designs. Thus it can provide a solid basis for physics studies.

In figures 6–9 I give a graphical impression of the quality of the fit, which appears to be as good as one could reasonably expect for a simple *ansatz* like (12). Note that the histograms have non-equidistant bins and that the resulting Jacobians have not been removed. Therefore the bin contents falls off at the singularities, as discussed above.

The errors used for the least-square fit had to be taken from a Monte Carlo (MC) study. **Guinea-Pig** only provides the \sqrt{n} from Poissonian statistics for each bin, but the error accumulation during tracking the particles through phase space is not available. The MC studies shows that the latter error dominates the former, but appears to be reasonably Gaussian. A complete MC study of all parameter sets is computationally expensive (more than a week of processor time on a fast SGI). From an exemplary MC study of a few parameter sets, it appears that the errors can be described reasonably well by rescaling the Poissonian error in each bin with appropriate factors for electrons/positrons and photons and for continuum and delta. This procedure has been adopted.

The $\chi^2/\text{d.o.f.}$'s of the fits are less than $\mathcal{O}(10)$. The simple *ansatz* (12) is therefore very satisfactory. In fact, trying to improve the ad-hoc factorized Beta distributions by the better motivated approximations from [7] or [16], it turns out [17] that (12) provides a significantly better fit of the results of the simulations. The price to pay is that the parameters in (12) have no direct

physical interpretation.

5.1.2 Generators

For this version of the parameterizations we need a fast generator of Beta distributions:

$$\beta^{a,b}(x) \propto x^{a-1}(1-x)^{b-1} \quad (14)$$

This problem has been studied extensively and we can use a published algorithm [18] that is guaranteed to be very fast for all a, b such that $0 < a \leq 1 \leq b$, which turns out to be always the case (cf. tables 5, 6, and 7).

5.2 Future Versions

There are two ways in which the parameterizations can be improved:

more complicated functions: the factorized fits can only be improved marginally by adding more positive semi-definite factors to (12). More improvement is possible by using sums of functions, but in this case, the best fits violate the positivity requirement and have to be discarded.

correlations: the parameterization in section 5.1 is factorized. While this is a good approximation, the simulations nevertheless show correlations among x_1 and x_2 . These correlations can be included in a future version.

interpolation: the parameterization in section 5.1 is based on fitting the simulation results by simple functions. Again, this appears to be a good approximation. But such fits can not uncover any fine structure of the distributions. Therefore it will be worthwhile to study interpolations of the simulation results in the future. A proper interpolation of results with statistical errors is however far from trivial: straightforward polynomial or spline interpolations will be oscillatory and violate the positivity requirement. Smoothing algorithms have to be investigated in depth before such a parameterization can be released.

other simulations: besides [5], other simulation codes are invited to contribute their results for inclusion in the `Circe1` library.

6 Implementation of circe1

```
29 <circe1.f90 29>≡
    ! circe1.f90 -- canonical beam spectra for linear collider physics
    ! $Id: circe1.nw 5607 2014-03-31 09:28:28Z ohl $
    <Copyleft notice 30a>
    <Main module 30c>
```

```

30a <Copyleft notice 30a>≡
!
! Copyright (C) 1999-2014 by
!   Wolfgang Kilian <kilian@physik.uni-siegen.de>
!   Thorsten Ohl <ohl@physik.uni-wuerzburg.de>
!   Juergen Reuter <juergen.reuter@desy.de>
!   Christian Speckner <cnspeckn@googlemail.com>
!
! WHIZARD is free software; you can redistribute it and/or modify it
! under the terms of the GNU General Public License as published by
! the Free Software Foundation; either version 2, or (at your option)
! any later version.
!
! WHIZARD is distributed in the hope that it will be useful, but
! WITHOUT ANY WARRANTY; without even the implied warranty of
! MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
! GNU General Public License for more details.
!
! You should have received a copy of the GNU General Public License
! along with this program; if not, write to the Free Software
! Foundation, Inc., 675 Mass Ave, Cambridge, MA 02139, USA.
!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
! This file has been stripped of most comments. For documentation, refer
! to the source 'circe1.nw'

```

Now we can move on to the implementation.

6.1 Symbolic Constants

The file `circe.h` contains symbolic names for various magic constants used by `Circe1`:

```

30b <circe.h 30b>≡
  c circe.h -- canonical beam spectra for linear collider physics
  c $Id: circe1.nw 5607 2014-03-31 09:28:28Z ohl $
Uses circe 31b.

30c <Main module 30c>≡
  module circe1
    use kinds

    implicit none
    private
    <Public subroutines 31a>

    <Public types 79e>

    <Particle codes 11b>
    <Accelerator codes 13a>
    <Private parameters 37c>

```

```

integer, parameter, public :: MAGIC0 = 19040616
real(kind=double), parameter :: KIREPS = 1D-6

<Declaration: circe1 parameters 32d>

type(circe1_params_t), public, save :: circe1_params

<Abstract types 79f>

<Abstract interfaces 79d>

contains

  <Module subroutines 31b>
end module circe1

```

6.2 Distributions

6.2.1 Version 1

We start with a convenience function which dispatches over the valid particle types. The hardest part is of course to avoid typos in such trivial functions ...

```

31a <Public subroutines 31a>≡
    public :: circe
    Uses circe 31b.

31b <Module subroutines 31b>≡
    function circe (x1, x2, p1, p2)
    real(kind=double) :: x1, x2
    integer :: p1, p2
    real(kind=double) :: circe
    <Initialization check 32h>
    circe = -1.0
    if (abs(p1) .eq. C1_ELECTRON) then
        if (abs(p2) .eq. C1_ELECTRON) then
            circe = circee (x1, x2)
        else if (p2 .eq. C1_PHOTON) then
            circe = circeg (x1, x2)
        end if
    else if (p1 .eq. C1_PHOTON) then
        if (abs(p2) .eq. C1_ELECTRON) then
            circe = circeg (x2, x1)
        else if (p2 .eq. C1_PHOTON) then
            circe = circgg (x1, x2)
        end if
    end if
end function circe

```

Defines:

circe, used in chunks 11a, 31, 30b, 31a, 87b, 93b, 110b, and 115e.
 Uses C1_ELECTRON 11b, C1_PHOTON 11b, circee 41h, circeg 42c, and circgg 43c.

32a *<Public subroutines 31a>+≡*
 public :: circes

Uses circes 32b.

32b *<Module subroutines 31b>+≡*
 subroutine circes (xx1m, xx2m, xroots, xacc, xver, xrev, xchat)
 real(kind=double) :: xx1m, xx2m, xroots
 integer :: xacc, xver, xrev, xchat
<Local variables for circes 33c>
<Initializations for circes 35b>
 if (circe1_params%magic .ne. 19040616) then
 circe1_params%magic = 19040616
<Initialize circe1 parameters 33a>
 end if
<Update circe1 parameters 33b>
<formats for circes 38d>
 end subroutine circes

Defines:

circes, used in chunks 32b, 12b, 17b, 32b, 21c, 32, 35f, 87b, and 91b.

32c *<Public subroutines 31a>+≡*
 public :: circe1_params_t

32d *<Declaration: circe1 parameters 32d>≡*
 type :: circe1_params_t
<8-byte aligned part of circe1 parameters 32e>
<4-byte aligned part of circe1 parameters 32f>
 end type circe1_params_t

32e *<8-byte aligned part of circe1 parameters 32e>≡*
 real(kind=double) :: x1m = 0d0
 real(kind=double) :: x2m = 0d0
 real(kind=double) :: roots = 500D0

32f *<4-byte aligned part of circe1 parameters 32f>≡*
 integer :: acc = TESLA
 integer :: ver = 0
 integer :: rev = 0
 integer :: chat = 1

Uses TESLA 13a.

Instead of using fragile block data subroutines, we use a magic number to tag circe1_params as initialized:

32g *<4-byte aligned part of circe1 parameters 32f>+≡*
 integer :: magic

Since negative values are no updated, we can call circes with all negative variables to ensure initialization:

32h *<Initialization check 32h>≡*
 if (circe1_params%magic .ne. MAGIC0) then
 call circes (-1d0, -1d0, -1d0, -1, -1, -1, -1)

```

endif
Uses circes 32b.
33a <Initialize circe1 parameters 33a>≡
    circe1_params%x1m = 0d0
    circe1_params%x2m = 0d0
    circe1_params%roots = 500D0
    circe1_params%acc = TESLA
    circe1_params%ver = 0
    circe1_params%rev = 0
    circe1_params%chat = 1
    if (xchat .ne. 0) then
        call circem ('MESSAGE', 'starting up ...')
        call circem ('MESSAGE', &
            '$Id: circe1.nw 5607 2014-03-31 09:28:28Z ohl $')
    endif
Uses TESLA 13a and circem 87a.
33b <Update circe1 parameters 33b>≡
    if ((xchat .ge. 0) .and. (xchat .ne. circe1_params%chat)) then
        circe1_params%chat = xchat
        if (circe1_params%chat .ge. 1) then
            write (msgbuf, 1000) 'chat', circe1_params%chat
1000    format ('updating ', A, ''' to ', I2)
            call circem ('MESSAGE', msgbuf)
        endif
    else
        if (circe1_params%chat .ge. 2) then
            write (msgbuf, 1100) 'chat', circe1_params%chat
1100    format ('keeping ', A, ''' at ', I2)
            call circem ('MESSAGE', msgbuf)
        endif
    endif
Uses circem 87a.
33c <Local variables for circes 33c>≡
    character(len=60) :: msgbuf
33d <Update circe1 parameters 33b>+=
    if ((xx1m .ge. 0d0) .and. (xx1m .ne. circe1_params%x1m)) then
        circe1_params%x1m = xx1m
        if (circe1_params%chat .ge. 1) then
            write (msgbuf, 1001) 'x1min', circe1_params%x1m
1001    format ('updating ', A, ''' to ', E12.4)
            call circem ('MESSAGE', msgbuf)
        endif
    else
        if (circe1_params%chat .ge. 2) then
            write (msgbuf, 1101) 'x1min', circe1_params%x1m
1101    format ('keeping ', A, ''' at ', E12.4)
            call circem ('MESSAGE', msgbuf)
        endif
    endif

```

```

endif
Uses circem 87a.
34a <Update circe1 parameters 33b>+≡
    if ((xx2m .ge. 0d0) .and. (xx2m .ne. circe1_params%x2m)) then
        circe1_params%x2m = xx2m
        if (circe1_params%chat .ge. 1) then
            write (msgbuf, 1001) 'x2min', circe1_params%x2m
            call circem ('MESSAGE', msgbuf)
        endif
    else
        if (circe1_params%chat .ge. 2) then
            write (msgbuf, 1101) 'x2min', circe1_params%x2m
            call circem ('MESSAGE', msgbuf)
        endif
    endif
Uses circem 87a.
34b <Update circe1 parameters 33b>+≡
    if ((xroots .ge. 0d0) .and. (xroots .ne. circe1_params%roots)) then
        circe1_params%roots = xroots
        if (circe1_params%chat .ge. 1) then
            write (msgbuf, 1002) 'roots', circe1_params%roots
1002     format ('updating ', A, ''' to ', F6.1)
            call circem ('MESSAGE', msgbuf)
        endif
    else
        if (circe1_params%chat .ge. 2) then
            write (msgbuf, 1102) 'roots', circe1_params%roots
1102     format ('keeping ', A, ''' at ', F6.1)
            call circem ('MESSAGE', msgbuf)
        endif
    endif
Uses circem 87a.
34c <Update circe1 parameters 33b>+≡
    if ((xacc .ge. 0) .and. (xacc .ne. circe1_params%acc)) then
        if ((xacc .ge. 1) .and. (xacc .le. NACC)) then
            circe1_params%acc = xacc
            if (circe1_params%chat .ge. 1) then
                write (msgbuf, 1003) 'acc', accnam(circe1_params%acc)
1003     format ('updating ', A, ''' to ', A)
                call circem ('MESSAGE', msgbuf)
            endif
        else
            write (msgbuf, 1203) xacc
1203     format ('invalid 'acc': ', I8)
            call circem ('ERROR', msgbuf)
            write (msgbuf, 1103) 'acc', accnam(circe1_params%acc)
1103     format ('keeping ', A, ''' at ', A)
            call circem ('MESSAGE', msgbuf)
        endif
    endif

```

```

        endif
    else
        if (circe1_params%chat .ge. 2) then
            write (msgbuf, 1003) 'acc', accnam(circe1_params%acc)
            call circem ('MESSAGE', msgbuf)
        endif
    endif
    if ((circe1_params%acc .eq. SBNDEE) .or. (circe1_params%acc .eq. TESLEE) &
        .or. (circe1_params%acc .eq. XBNDEE)) then
        <Warn that no parameter set has been endorsed for  $e^-e^-$  yet 36a>
    endif
    Uses NACC 13b, SBNDEE 13a, TESLEE 13a, XBNDEE 13a, and circem 87a.

35a <Local variables for circes 33c>+≡
    <Declaration of accnam 35c>

35b <Initializations for circes 35b>≡
    <Initialization of accnam 35d>

35c <Declaration of accnam 35c>≡
    character(len=6), dimension(NACC) :: accnam
    Uses NACC 13b.

35d <Initialization of accnam 35d>≡
    data accnam(SBAND) /'SBAND'/
    data accnam(TESLA) /'TESLA'/
    data accnam(JLCNLC) /'JLCNLC'/
    data accnam(SBNDEE) /'SBNDEE'/
    data accnam(TESLEE) /'TESLEE'/
    data accnam(XBNDEE) /'XBNDEE'/
    data accnam(NLCH) /'NLC H'/
    data accnam(ILC) /'ILC'/
    data accnam(CLIC) /'CLIC'/
    Uses CLIC 13a, ILC 13a, JLCNLC 13a, SBAND 13a, SBNDEE 13a, TESLA 13a, TESLEE 13a,
    and XBNDEE 13a.

35e <Public subroutines 31a>+≡
    public :: circex
    Uses circex 35f.

35f <Module subroutines 31b>+≡
    subroutine circex (xx1m, xx2m, xroots, cacc, xver, xrev, xchat)
        real(kind=double) :: xx1m, xx2m, xroots
        character(*) :: cacc
        integer :: xver, xrev, xchat
        integer :: xacc, i
        <Accelerator codes 13a>
        <Declaration of accnam 35c>
        <Initialization of accnam 35d>
        xacc = -1
        do i = 1, NACC
            if (trim (accnam(i)) == trim (cacc)) then
                xacc = i
            end if
        end do
    end subroutine

```

```

        end if
    end do
    call circex (xx1m, xx2m, xroots, xacc, xver, xrev, xchat)
end subroutine circex

```

Defines:

circex, used in chunk 35e.

Uses NACC 13b and circex 32b.

36a *<Warn that no parameter set has been endorsed for e^-e^- yet 36a>*≡

```

    call circem ('WARNING', '*****')
    call circem ('WARNING', '* The accelerator parameters have *')
    call circem ('WARNING', '* not been endorsed for use in *')
    call circem ('WARNING', '* an e-e- collider yet!!! *')
    call circem ('WARNING', '*****')

```

Uses circem 87a.

36b *<Update circe1 parameters 33b>*+≡

```

    if (xver .ge. 0) then
        circe1_params%ver = xver
        if (circe1_params%chat .ge. 1) then
            write (msgbuf, 1000) 'ver', circe1_params%ver
            call circem ('MESSAGE', msgbuf)
        endif
    else
        if (circe1_params%chat .ge. 2) then
            write (msgbuf, 1100) 'ver', circe1_params%ver
            call circem ('MESSAGE', msgbuf)
        endif
    endif
endif

```

Uses circem 87a.

36c *<Update circe1 parameters 33b>*+≡

```

    if ((xrev .ge. 0) .and.(xrev .ne. circe1_params%rev)) then
        circe1_params%rev = xrev
        if (circe1_params%chat .ge. 1) then
            write (msgbuf, 1004) 'rev', circe1_params%rev
1004         format ('updating ', A, ''' to ', I8)
            call circem ('MESSAGE', msgbuf)
        endif
    else
        if (circe1_params%chat .ge. 2) then
            write (msgbuf, 1104) 'rev', circe1_params%rev
1104         format ('keeping ', A, ''' at ', I8)
            call circem ('MESSAGE', msgbuf)
        endif
    endif
endif

```

Uses circem 87a.

Versions 3 and 4 are identical to version 1, except for TESLA at 800 GeV.

36d *<Update circe1 parameters 33b>*+≡

```

    ver34 = 0
    if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then

```

```

    <Update version 1 derived parameters in circe1 parameters 37d>
      else if ((circe1_params%ver .eq. 3) .or. (circe1_params%ver .eq. 4)) then
        ver34 = circe1_params%ver
        circe1_params%ver = 1
    <Update version 3 and 4 derived parameters in circe1 parameters 51a>
      else if (circe1_params%ver .eq. 5) then
        circe1_params%ver = 1
    <Update version 5 derived parameters in circe1 parameters 53a>
      else if (circe1_params%ver .eq. 6) then
        circe1_params%ver = 1
    <Update version 6 derived parameters in circe1 parameters 54d>
      else if (circe1_params%ver .eq. 7) then
        circe1_params%ver = 1
    <Update version 7 derived parameters in circe1 parameters 57a>
      else if (circe1_params%ver .eq. 8) then
        circe1_params%ver = 1
    <Update version 8 derived parameters in circe1 parameters 61b>
      else if (circe1_params%ver .eq. 9) then
        circe1_params%ver = 1
    <Update version 9 derived parameters in circe1 parameters 63b>
      else if (circe1_params%ver .eq. 10) then
        circe1_params%ver = 1
    <Update version 10 derived parameters in circe1 parameters 69a>
    <else handle invalid versions 37b>
37a <Local variables for circes 33c>+=
      integer :: ver34
37b <else handle invalid versions 37b>=
      else if (circe1_params%ver .eq. 2) then
    <Version 2 has been retired 50c>
      else if (circe1_params%ver .gt. 10) then
        call circem ('PANIC', 'versions >10 not available yet')
        return
      else
        call circem ('PANIC', 'version must be positive')
        return
      end if
    Uses circem 87a.
37c <Private parameters 37c>=
      integer :: e, r, ehi, elo
37d <Update version 1 derived parameters in circe1 parameters 37d>=
      if (circe1_params%rev .eq. 0) then
        r = 0
      elseif (circe1_params%rev .ge. 19970417) then
        r = 5
      elseif (circe1_params%rev .ge. 19960902) then
        r = 4
      elseif (circe1_params%rev .ge. 19960729) then
        r = 3

```

```

elseif (circe1_params%rev .ge. 19960711) then
  r = 2
elseif (circe1_params%rev .ge. 19960401) then
  r = 1
elseif (circe1_params%rev .lt. 19960401) then
  call circem ('ERROR', &
    'no revision of version 1 before 96/04/01 available')
  call circem ('MESSAGE', 'falling back to default')
  r = 1
endif
if (circe1_params%chat .ge. 2) then
  write (msgbuf, 2000) circe1_params%rev, r
2000  format ('mapping date ', I8, ' to revision index ', I2)
  call circem ('MESSAGE', msgbuf)
endif
Uses circem 87a.

38a  <Log revision mapping 38a>≡
      if (circe1_params%chat .ge. 2) then
        write (msgbuf, 2000) circe1_params%rev, r
        call circem ('MESSAGE', msgbuf)
      endif
Uses circem 87a.

38b  <Update version 1 derived parameters in circe1 parameters 37d>+≡
      <Map roots to e 38c>

38c  <Map roots to e 38c>≡
      if (circe1_params%roots .eq. 350d0) then
        e = GEV350
      else if ((circe1_params%roots .ge. 340d0) .and. (circe1_params%roots .le. 370d0))
        write (msgbuf, 2001) circe1_params%roots, 350d0
        call circem ('MESSAGE', msgbuf)
        e = GEV350
Uses circem 87a.

38d  <formats for circes 38d>≡
      2001 format ('treating energy ', F6.1, 'GeV as ', F6.1, 'GeV')

38e  <Map roots to e 38c>+≡
      else if (circe1_params%roots .eq. 500d0) then
        e = GEV500
      else if ((circe1_params%roots .ge. 480d0) .and. (circe1_params%roots .le. 520d0))
        write (msgbuf, 2001) circe1_params%roots, 500d0
        call circem ('MESSAGE', msgbuf)
        e = GEV500
      else if (circe1_params%roots .eq. 800d0) then
        e = GEV800
      else if ((circe1_params%roots .ge. 750d0) .and. (circe1_params%roots .le. 850d0))
        write (msgbuf, 2001) circe1_params%roots, 800d0
        call circem ('MESSAGE', msgbuf)
        e = GEV800

```

```

else if (circe1_params%roots .eq. 1000d0) then
  e = TEV1
else if ((circe1_params%roots .ge. 900d0) .and. (circe1_params%roots .le. 1100d0))
  write (msgbuf, 2001) circe1_params%roots, 1000d0
  call circem ('MESSAGE', msgbuf)
  e = TEV1
else if (circe1_params%roots .eq. 1600d0) then
  e = TEV16
else if ((circe1_params%roots .ge. 1500d0) .and. (circe1_params%roots .le. 1700d0))
  write (msgbuf, 2001) circe1_params%roots, 1600d0
  call circem ('MESSAGE', msgbuf)
  e = TEV16

```

Uses circem 87a.

39a *<Map roots to e 38c>+≡*

```

else
  call circem ('ERROR', &
    'only ROOTS = 350, 500, 800, 1000 and 1600GeV available')
  call circem ('MESSAGE', 'falling back to 500GeV')
  e = GEV500
endif

```

Uses circem 87a.

39b *<Update version 1 derived parameters in circe1 parameters 37d>+≡*

```

if (xallum(e,circe1_params%acc,r) .lt. 0d0) then
  write (msgbuf, 2002) circe1_params%roots, accnam(circe1_params%acc), r
  call circem ('ERROR', msgbuf)
  call circem ('MESSAGE', 'falling back to 500GeV')
  e = GEV500
end if

```

<Log energy mapping 39d>

Uses circem 87a.

39c *<formats for circes 38d>+≡*

```

2002 format ('energy ', F6.1, ' not available for ', A6,' in revision ', I2)

```

39d *<Log energy mapping 39d>≡*

```

if (circe1_params%chat .ge. 2) then
  if (e .ge. GEV090) then
    write (msgbuf, 2003) circe1_params%roots, e
    call circem ('MESSAGE', msgbuf)
  else if (elo .ge. GEV090 .and. ehi .ge. GEV090) then
    write (msgbuf, 2013) circe1_params%roots, elo, ehi
    call circem ('MESSAGE', msgbuf)
  end if
endif

```

Uses circem 87a.

39e *<formats for circes 38d>+≡*

```

2003 format ('mapping energy ', F6.1, ' to energy index ', I2)
2013 format ('mapping energy ', F6.1, ' to energy indices ', I2, ' and ', I2)

```

The energies 250 GeV, 1.2 TeV and 1.5 TeV were entered late into the game by the SLAC people. And, of course, 200 GeV and 230 GeV only appeared even much later

```

40a <Local variables for circes 33c>+≡
      integer, parameter :: EINVALID = -2
      integer, parameter :: GEV090 = -1
      integer, parameter :: GEV170 = 0
      integer, parameter :: GEV350 = 1
      integer, parameter :: GEV500 = 2
      integer, parameter :: GEV800 = 3
      integer, parameter :: TEV1 = 4
      integer, parameter :: TEV16 = 5
      integer, parameter :: GEV250 = 6
      integer, parameter :: TEV12 = 7
      integer, parameter :: TEV15 = 8
      integer, parameter :: GEV200 = 9
      integer, parameter :: GEV230 = 10
      integer, parameter :: A1NEGY = 5
      integer, parameter :: A1NREV = 5
      integer :: i

40b <8-byte aligned part of circe1 parameters 32e>+≡
      real(kind=double) :: lumi
      real(kind=double) :: a1(0:7)

40c <Update version 1 derived parameters in circe1 parameters 37d>+≡
      circe1_params%lumi = xallum(e,circe1_params%acc,r)
      do i = 0, 7
         circe1_params%a1(i) = xa1(i,e,circe1_params%acc,r)
      end do

40d <Local variables for circes 33c>+≡
      real(kind=double), dimension(A1NEGY,NACC,0:A1NREV), save :: xallum = 0
      real(kind=double), dimension(0:7,A1NEGY,NACC,0:A1NREV), save :: xa1 = 0

```

Uses NACC 13b.

Revision 1. The mother of all revisions.

```

40e <Initializations for circes 35b>+≡
      xallum(GEV500,SBAND,1) = 5.212299E+01
      xa1(0:7,GEV500,SBAND,1) = (/ &
         .39192E+00, .66026E+00, .11828E+02,-.62543E+00, &
         .52292E+00,-.69245E+00, .14983E+02, .65421E+00 /)
      xallum(GEV500,TESLA,1) = 6.066178E+01
      xa1(0:7,GEV500,TESLA,1) = (/ &
         .30196E+00, .12249E+01, .21423E+02,-.57848E+00, &
         .68766E+00,-.69788E+00, .23121E+02, .78399E+00 /)
      xallum(GEV500,XBAND,1) = 5.884699E+01
      xa1(0:7,GEV500,XBAND,1) = (/ &
         .48594E+00, .52435E+00, .83585E+01,-.61347E+00, &
         .30703E+00,-.68804E+00, .84109E+01, .44312E+00 /)

```

Uses SBAND 13a, TESLA 13a, and XBAND 13a.

41a *<Initializations for circes 35b>+≡*
`xa1lum(TEV1,SBAND,1) = 1.534650E+02`
`xa1(0:7,TEV1,SBAND,1) = (/ &`
`.24399E+00, .87464E+00, .66751E+01,-.56808E+00, &`
`.59295E+00,-.68921E+00, .94232E+01, .83351E+00 /)`
`xa1lum(TEV1,TESLA,1) = 1.253381E+03`
`xa1(0:7,TEV1,TESLA,1) = (/ &`
`.39843E+00, .70097E+00, .11602E+02,-.61061E+00, &`
`.40737E+00,-.69319E+00, .14800E+02, .51382E+00 /)`
`xa1lum(TEV1,XBAND,1) = 1.901783E+02`
`xa1(0:7,TEV1,XBAND,1) = (/ &`
`.32211E+00, .61798E+00, .28298E+01, -.54644E+00, &`
`.45674E+00, -.67301E+00, .41703E+01, .74536E+00 /)`
Uses SBAND 13a, TESLA 13a, and XBAND 13a.
Unavailable

41b *<Initializations for circes 35b>+≡*
`xa1lum(GEV350,1:NACC,1) = NACC * (-1d0)`
`xa1lum(GEV800,1:NACC,1) = NACC * (-1d0)`
Uses NACC 13b.
Unavailable as well

41c *<Initializations for circes 35b>+≡*
`xa1lum(GEV500,SBNDDEE:NACC,1) = 4 * (-1d0)`
`xa1lum(TEV1,SBNDDEE:NACC,1) = 4 * (-1d0)`
Uses NACC 13b and SBNDDEE 13a.
No 1.6TeV parameters in this revision

41d *<Initializations for circes 35b>+≡*
`xa1lum(TEV16,1:NACC,1) = 7 * (-1d0)`
Uses NACC 13b.

41e *<Public subroutines 31a>+≡*
`public :: circel`
Uses circel 41f.

41f *<Module subroutines 31b>+≡*
`subroutine circel (l)`
`real(kind=double), intent(out) :: l`
`l = circel_params%lumi`
`end subroutine circel`

Defines:
 circel, used in chunks 41f, 12a, and 41e.

41g *<Public subroutines 31a>+≡*
`public :: circee`
Uses circee 41h.

41h *<Module subroutines 31b>+≡*
`function circee (x1, x2)`
`real(kind=double) :: x1, x2`
`real(kind=double) :: circee`

```

    real(kind=double) :: d1, d2
    <Initialization check 32h>
    circee = -1.0
    if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
    <Calculate version 1 of the  $e^+e^-$  distribution 42a>
    <else handle invalid versions 37b>
    end function circee

```

Defines:

`circee`, used in chunks 14–16, 31b, 41g, and 42a.

Uses `d1` 16a and `d2` 16c.

The first version of the parametrization is factorized

$$D_{p_1 p_2}^{\alpha_1 \rho}(x_1, x_2, s) = d_{p_1}^{\alpha_1 \rho}(x_1) d_{p_2}^{\alpha_1 \rho}(x_2) \quad (15)$$

where the distributions are

$$d_{e^\pm}^{\alpha_1 \rho}(x) = a_0^{\alpha_1 \rho} \delta(1-x) + a_1^{\alpha_1 \rho} x^{a_2^{\alpha_1 \rho}} (1-x)^{a_3^{\alpha_1 \rho}} \quad (16)$$

$$d_\gamma(x) = a_4^{\alpha_1 \rho} x^{a_5^{\alpha_1 \rho}} (1-x)^{a_6^{\alpha_1 \rho}} \quad (17)$$

42a <Calculate version 1 of the e^+e^- distribution 42a>≡

```

    if (x1 .eq. 1d0) then
        d1 = circe1_params%a1(0)
    elseif (x1 .lt. 1d0 .and. x1 .gt. 0d0) then
        d1 = circe1_params%a1(1) * x1**circe1_params%a1(2) * (1d0 - x1)**circe1_params%
    elseif (x1 .eq. -1d0) then
        d1 = 1d0 - circe1_params%a1(0)
    else
        d1 = 0d0
    endif
    if (x2 .eq. 1d0) then
        d2 = circe1_params%a1(0)
    elseif (x2 .lt. 1d0 .and. x2 .gt. 0d0) then
        d2 = circe1_params%a1(1) * x2**circe1_params%a1(2) * (1d0 - x2)**circe1_params%
    elseif (x2 .eq. -1d0) then
        d2 = 1d0 - circe1_params%a1(0)
    else
        d2 = 0d0
    endif
    circee = d1 * d2

```

Uses `circee` 41h, `d1` 16a, and `d2` 16c.

42b <Public subroutines 31a>+≡

```

    public :: circeg

```

Uses `circeg` 42c.

42c <Module subroutines 31b>+≡

```

    function circeg (x1, x2)
    real(kind=double) :: x1, x2
    real(kind=double) :: circeg
    real(kind=double) :: d1, d2

```

```

    <Initialization check 32h>
    circeg = -1.0
    if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
    <Calculate version 1 of the  $e^\pm\gamma$  distribution 43a>
    <else handle invalid versions 37b>
    end function circeg

```

Defines:

circeg, used in chunks 14, 31b, 42b, and 43a.

Uses d1 16a and d2 16c.

```

43a <Calculate version 1 of the  $e^\pm\gamma$  distribution 43a>≡
    if (x1 .eq. 1d0) then
        d1 = circe1_params%a1(0)
    else if (x1 .lt. 1d0 .and. x1 .gt. 0d0) then
        d1 = circe1_params%a1(1) * x1**circe1_params%a1(2) * (1d0 - x1)**circe1_params%a1(3)
    else if (x1 .eq. -1d0) then
        d1 = 1d0 - circe1_params%a1(0)
    else
        d1 = 0d0
    end if
    if (x2 .lt. 1d0 .and. x2 .gt. 0d0) then
        d2 = circe1_params%a1(4) * x2**circe1_params%a1(5) * (1d0 - x2)**circe1_params%a1(6)
    else if (x2 .eq. -1d0) then
        d2 = circe1_params%a1(7)
    else
        d2 = 0d0
    end if
    circeg = d1 * d2

```

Uses circeg 42c, d1 16a, and d2 16c.

```

43b <Public subroutines 31a>+≡
    public :: circgg

```

Uses circgg 43c.

```

43c <Module subroutines 31b>+≡
    function circgg (x1, x2)
        real(kind=double) :: x1, x2
        real(kind=double) :: circgg
        real(kind=double) :: d1, d2
    <Initialization check 32h>
    circgg = -1.0
    if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
    <Calculate version 1 of the  $\gamma\gamma$  distribution 43d>
    <else handle invalid versions 37b>
    end function circgg

```

Defines:

circgg, used in chunks 14, 31b, 43, and 81a.

Uses d1 16a and d2 16c.

```

43d <Calculate version 1 of the  $\gamma\gamma$  distribution 43d>≡

```

```

if (x1 .lt. 1d0 .and. x1 .gt. 0d0) then
  d1 = circe1_params%a1(4) * x1**circe1_params%a1(5) * (1d0 - x1)**circe1_params%
elseif (x1 .eq. -1d0) then
  d1 = circe1_params%a1(7)
else
  d1 = 0d0
endif
if (x2 .lt. 1d0 .and. x2 .gt. 0d0) then
  d2 = circe1_params%a1(4) * x2**circe1_params%a1(5) * (1d0 - x2)**circe1_params%
elseif (x2 .eq. -1d0) then
  d2 = circe1_params%a1(7)
else
  d2 = 0d0
endif
circgg = d1 * d2

```

Uses circgg 43c, d1 16a, and d2 16c.

Revision 2. New Tesla parameters, including 350 GeV and 800 GeV.

44 *(Initializations for circes 35b)+≡*

```

xa1lum(GEV500,SBAND,2) = .31057E+02
xa1(0:7,GEV500,SBAND,2) = (/ &
  .38504E+00, .79723E+00, .14191E+02,-.60456E+00, &
  .53411E+00,-.68873E+00, .15105E+02, .65151E+00 /)
xa1lum(TEV1,SBAND,2) = .24297E+03
xa1(0:7,TEV1,SBAND,2) = (/ &
  .24374E+00, .89466E+00, .70242E+01,-.56754E+00, &
  .60910E+00,-.68682E+00, .96083E+01, .83985E+00 /)
xa1lum(GEV350,TESLA,2) = .73369E+02
xa1(0:7,GEV350,TESLA,2) = (/ &
  .36083E+00, .12819E+01, .37880E+02,-.59492E+00, &
  .69109E+00,-.69379E+00, .40061E+02, .65036E+00 /)
xa1lum(GEV500,TESLA,2) = .10493E+03
xa1(0:7,GEV500,TESLA,2) = (/ &
  .29569E+00, .11854E+01, .21282E+02,-.58553E+00, &
  .71341E+00,-.69279E+00, .24061E+02, .77709E+00 /)
xa1lum(GEV800,TESLA,2) = .28010E+03
xa1(0:7,GEV800,TESLA,2) = (/ &
  .22745E+00, .11265E+01, .10483E+02,-.55711E+00, &
  .69579E+00,-.69068E+00, .13093E+02, .89605E+00 /)
xa1lum(TEV1,TESLA,2) = .10992E+03
xa1(0:7,TEV1,TESLA,2) = (/ &
  .40969E+00, .66105E+00, .11972E+02,-.62041E+00, &
  .40463E+00,-.69354E+00, .14669E+02, .51281E+00 /)
xa1lum(GEV500,XBAND,2) = .35689E+02
xa1(0:7,GEV500,XBAND,2) = (/ &
  .48960E+00, .46815E+00, .75249E+01,-.62769E+00, &
  .30341E+00,-.68754E+00, .85545E+01, .43453E+00 /)
xa1lum(TEV1,XBAND,2) = .11724E+03
xa1(0:7,TEV1,XBAND,2) = (/ &
  .31939E+00, .62415E+00, .30763E+01,-.55314E+00, &

```

.45634E+00,-.67089E+00, .41529E+01, .73807E+00 /)

Uses SBAND 13a, TESLA 13a, and XBAND 13a.

Unavailable

45a *<Initializations for circes 35b>+≡*
`xa1lum(GEV350,SBAND,2) = -1d0`
`xa1lum(GEV350,XBAND,2) = -1d0`
`xa1lum(GEV800,SBAND,2) = -1d0`
`xa1lum(GEV800,XBAND,2) = -1d0`

Uses SBAND 13a and XBAND 13a.

Unavailable as well

45b *<Initializations for circes 35b>+≡*
`xa1lum(GEV350,SBNDDEE:NACC,2) = 4 * (-1d0)`
`xa1lum(GEV500,SBNDDEE:NACC,2) = 4 * (-1d0)`
`xa1lum(GEV800,SBNDDEE:NACC,2) = 4 * (-1d0)`
`xa1lum(TEV1,SBNDDEE:NACC,2) = 4 * (-1d0)`

Uses NACC 13b and SBNDDEE 13a.

No 1.6TeV parameters in this revision

45c *<Initializations for circes 35b>+≡*
`xa1lum(TEV16,1:NACC,2) = 7 * (-1d0)`

Uses NACC 13b.

Revision 3. Features:

- improved error estimates.
- cleaner fitting procedure, including delta function pieces.

45d *<Initializations for circes 35b>+≡*
`xa1lum(GEV500,SBAND,3) = .31469E+02`
`xa1(0:7,GEV500,SBAND,3) = (/ &`
`.38299E+00, .72035E+00, .12618E+02,-.61611E+00, &`
`.51971E+00,-.68960E+00, .15066E+02, .63784E+00 /)`
`xa1lum(TEV1,SBAND,3) = .24566E+03`
`xa1(0:7,TEV1,SBAND,3) = (/ &`
`.24013E+00, .95763E+00, .69085E+01,-.55151E+00, &`
`.59497E+00,-.68622E+00, .94494E+01, .82158E+00 /)`
`xa1lum(GEV350,TESLA,3) = .74700E+02`
`xa1(0:7,GEV350,TESLA,3) = (/ &`
`.34689E+00, .12484E+01, .33720E+02,-.59523E+00, &`
`.66266E+00,-.69524E+00, .38488E+02, .63775E+00 /)`
`xa1lum(GEV500,TESLA,3) = .10608E+03`
`xa1(0:7,GEV500,TESLA,3) = (/ &`
`.28282E+00, .11700E+01, .19258E+02,-.58390E+00, &`
`.68777E+00,-.69402E+00, .23638E+02, .75929E+00 /)`
`xa1lum(GEV800,TESLA,3) = .28911E+03`
`xa1(0:7,GEV800,TESLA,3) = (/ &`
`.21018E+00, .12039E+01, .96763E+01,-.54024E+00, &`
`.67220E+00,-.69083E+00, .12733E+02, .87355E+00 /)`
`xa1lum(TEV1,TESLA,3) = .10936E+03`
`xa1(0:7,TEV1,TESLA,3) = (/ &`

```

        .41040E+00, .68099E+00, .11610E+02,-.61237E+00, &
        .40155E+00,-.69073E+00, .14698E+02, .49989E+00 /)
xa1lum(GEV500,XBAND,3) = .36145E+02
xa1(0:7,GEV500,XBAND,3) = (/ &
        .51285E+00, .45812E+00, .75135E+01,-.62247E+00, &
        .30444E+00,-.68530E+00, .85519E+01, .43062E+00 /)
xa1lum(TEV1,XBAND,3) = .11799E+03
xa1(0:7,TEV1,XBAND,3) = (/ &
        .31241E+00, .61241E+00, .29938E+01,-.55848E+00, &
        .44801E+00,-.67116E+00, .41119E+01, .72753E+00 /)

```

Uses SBAND 13a, TESLA 13a, and XBAND 13a.

Still unavailable

46a *⟨Initializations for circes 35b⟩+≡*

```

        xa1lum(GEV350,SBAND,3) = -1d0
        xa1lum(GEV350,XBAND,3) = -1d0
        xa1lum(GEV800,SBAND,3) = -1d0
        xa1lum(GEV800,XBAND,3) = -1d0

```

Uses SBAND 13a and XBAND 13a.

Unavailable as well

46b *⟨Initializations for circes 35b⟩+≡*

```

        xa1lum(GEV350,SBNDDEE:NACC,3) = 4 * (-1d0)
        xa1lum(GEV500,SBNDDEE:NACC,3) = 4 * (-1d0)
        xa1lum(GEV800,SBNDDEE:NACC,3) = 4 * (-1d0)
        xa1lum(TEV1,SBNDDEE:NACC,3) = 4 * (-1d0)

```

Uses NACC 13b and SBNDDEE 13a.

No 1.6TeV parameters in this revision

46c *⟨Initializations for circes 35b⟩+≡*

```

        xa1lum(TEV16,1:NACC,3) = 7 * (-1d0)

```

Uses NACC 13b.

Revision 4. Features:

- a bug in Guinea-Pig's synchrotron radiation spectrum has been fixed.

46d *⟨Initializations for circes 35b⟩+≡*

```

        xa1lum(GEV500,SBAND,4) = .31528E+02
        xa1(0:7,GEV500,SBAND,4) = (/ &
        .38169E+00, .73949E+00, .12543E+02,-.61112E+00, &
        .51256E+00,-.69009E+00, .14892E+02, .63314E+00 /)
        xa1lum(TEV1,SBAND,4) = .24613E+03
        xa1(0:7,TEV1,SBAND,4) = (/ &
        .24256E+00, .94117E+00, .66775E+01,-.55160E+00, &
        .57484E+00,-.68891E+00, .92271E+01, .81162E+00 /)
        xa1lum(GEV350,TESLA,4) = .74549E+02
        xa1(0:7,GEV350,TESLA,4) = (/ &
        .34120E+00, .12230E+01, .32932E+02,-.59850E+00, &
        .65947E+00,-.69574E+00, .38116E+02, .63879E+00 /)
        xa1lum(GEV500,TESLA,4) = .10668E+03
        xa1(0:7,GEV500,TESLA,4) = (/ &
        .28082E+00, .11074E+01, .18399E+02,-.59118E+00, &

```

```

        .68880E+00,-.69375E+00, .23463E+02, .76073E+00 /)
xa1lum(GEV800,TESLA,4) = .29006E+03
xa1(0:7,GEV800,TESLA,4) = (/ &
        .21272E+00, .11443E+01, .92564E+01,-.54657E+00, &
        .66799E+00,-.69137E+00, .12498E+02, .87571E+00 /)
xa1lum(TEV1,TESLA,4) = .11009E+03
xa1(0:7,TEV1,TESLA,4) = (/ &
        .41058E+00, .64745E+00, .11271E+02,-.61996E+00, &
        .39801E+00,-.69150E+00, .14560E+02, .49924E+00 /)
xa1lum(GEV500,XBAND,4) = .36179E+02
xa1(0:7,GEV500,XBAND,4) = (/ &
        .51155E+00, .43313E+00, .70446E+01,-.63003E+00, &
        .29449E+00,-.68747E+00, .83489E+01, .42458E+00 /)
xa1lum(TEV1,XBAND,4) = .11748E+03
xa1(0:7,TEV1,XBAND,4) = (/ &
        .32917E+00, .54322E+00, .28493E+01,-.57959E+00, &
        .39266E+00,-.68217E+00, .38475E+01, .68478E+00 /)

```

Uses SBAND 13a, TESLA 13a, and XBAND 13a.

Still unavailable

47a *⟨Initializations for circes 35b⟩*+≡

```

        xa1lum(GEV350,SBAND,4) = -1d0
        xa1lum(GEV350,XBAND,4) = -1d0
        xa1lum(GEV800,SBAND,4) = -1d0
        xa1lum(GEV800,XBAND,4) = -1d0

```

Uses SBAND 13a and XBAND 13a.

Unavailable as well

47b *⟨Initializations for circes 35b⟩*+≡

```

        xa1lum(GEV350,SBNDDEE:NACC,4) = 4 * (-1d0)
        xa1lum(GEV500,SBNDDEE:NACC,4) = 4 * (-1d0)
        xa1lum(GEV800,SBNDDEE:NACC,4) = 4 * (-1d0)
        xa1lum(TEV1,SBNDDEE:NACC,4) = 4 * (-1d0)

```

Uses NACC 13b and SBNDDEE 13a.

No 1.6TeV parameters in this revision

47c *⟨Initializations for circes 35b⟩*+≡

```

        xa1lum(TEV16,1:NACC,4) = 7 * (-1d0)

```

Uses NACC 13b.

Revision 5. Features:

- a bug in Guinea-Pig has been fixed.
- updated parameter sets

47d *⟨Initializations for circes 35b⟩*+≡

```

        xa1lum(GEV350,SBAND,5) = 0.21897E+02
        xa1(0:7,GEV350,SBAND,5) = (/ &
                0.57183E+00, 0.53877E+00, 0.19422E+02,-0.63064E+00, &
                0.49112E+00,-0.69109E+00, 0.24331E+02, 0.52718E+00 /)
        xa1lum(GEV500,SBAND,5) = 0.31383E+02
        xa1(0:7,GEV500,SBAND,5) = (/ &

```

```

0.51882E+00, 0.49915E+00, 0.11153E+02,-0.63017E+00, &
0.50217E+00,-0.69113E+00, 0.14935E+02, 0.62373E+00 /)
xa1lum(GEV800,SBAND,5) = 0.95091E+02
xa1(0:7,GEV800,SBAND,5) = (/ &
0.47137E+00, 0.46150E+00, 0.56562E+01,-0.61758E+00, &
0.46863E+00,-0.68897E+00, 0.85876E+01, 0.67577E+00 /)
xa1lum(TEV1,SBAND,5) = 0.11900E+03
xa1(0:7,TEV1,SBAND,5) = (/ &
0.43956E+00, 0.45471E+00, 0.42170E+01,-0.61180E+00, &
0.48711E+00,-0.68696E+00, 0.67145E+01, 0.74551E+00 /)
xa1lum(TEV16,SBAND,5) = 0.11900E+03
xa1(0:7,TEV16,SBAND,5) = (/ &
0.43956E+00, 0.45471E+00, 0.42170E+01,-0.61180E+00, &
0.48711E+00,-0.68696E+00, 0.67145E+01, 0.74551E+00 /)
xa1lum(GEV350,TESLA,5) = 0.97452E+02
xa1(0:7,GEV350,TESLA,5) = (/ &
0.39071E+00, 0.84996E+00, 0.17614E+02,-0.60609E+00, &
0.73920E+00,-0.69490E+00, 0.28940E+02, 0.77286E+00 /)
xa1lum(GEV500,TESLA,5) = 0.10625E+03
xa1(0:7,GEV500,TESLA,5) = (/ &
0.42770E+00, 0.71457E+00, 0.15284E+02,-0.61664E+00, &
0.68166E+00,-0.69208E+00, 0.24165E+02, 0.73806E+00 /)
xa1lum(GEV800,TESLA,5) = 0.17086E+03
xa1(0:7,GEV800,TESLA,5) = (/ &
0.36025E+00, 0.69118E+00, 0.76221E+01,-0.59440E+00, &
0.71269E+00,-0.69077E+00, 0.13117E+02, 0.91780E+00 /)
xa1lum(TEV1,TESLA,5) = 0.21433E+03
xa1(0:7,TEV1,TESLA,5) = (/ &
0.33145E+00, 0.67075E+00, 0.55438E+01,-0.58468E+00, &
0.72503E+00,-0.69084E+00, 0.99992E+01, 0.10112E+01 /)
xa1lum(TEV16,TESLA,5) = 0.34086E+03
xa1(0:7,TEV16,TESLA,5) = (/ &
0.49058E+00, 0.42609E+00, 0.50550E+01,-0.61867E+00, &
0.39225E+00,-0.68916E+00, 0.75514E+01, 0.58754E+00 /)
xa1lum(GEV350,XBAND,5) = 0.31901E+02
xa1(0:7,GEV350,XBAND,5) = (/ &
0.65349E+00, 0.31752E+00, 0.94342E+01,-0.64291E+00, &
0.30364E+00,-0.68989E+00, 0.11446E+02, 0.40486E+00 /)
xa1lum(GEV500,XBAND,5) = 0.36386E+02
xa1(0:7,GEV500,XBAND,5) = (/ &
0.65132E+00, 0.28728E+00, 0.69853E+01,-0.64440E+00, &
0.28736E+00,-0.68758E+00, 0.83227E+01, 0.41492E+00 /)
xa1lum(GEV800,XBAND,5) = 0.10854E+03
xa1(0:7,GEV800,XBAND,5) = (/ &
0.49478E+00, 0.36221E+00, 0.30116E+01,-0.61548E+00, &
0.39890E+00,-0.68418E+00, 0.45183E+01, 0.67243E+00 /)
xa1lum(TEV1,XBAND,5) = 0.11899E+03
xa1(0:7,TEV1,XBAND,5) = (/ &
0.49992E+00, 0.34299E+00, 0.26184E+01,-0.61584E+00, &
0.38450E+00,-0.68342E+00, 0.38589E+01, 0.67408E+00 /)

```

```

xaillum(TEV16,XBAND,5) = 0.13675E+03
xa1(0:7,TEV16,XBAND,5) = (/ &
    0.50580E+00, 0.30760E+00, 0.18339E+01,-0.61421E+00, &
    0.35233E+00,-0.68315E+00, 0.26708E+01, 0.67918E+00 /)

```

Uses SBAND 13a, TESLA 13a, and XBAND 13a.

Revision 0. Features:

- e^-e^- mode

49a *⟨Initializations for circes 35b⟩+≡*

```

xaillum(GEV500,SBNDDEE,0) = .92914E+01
xa1(0:7,GEV500,SBNDDEE,0) = (/ &
    .34866E+00, .78710E+00, .10304E+02,-.59464E+00, &
    .40234E+00,-.69741E+00, .20645E+02, .47274E+00 /)
xaillum(TEV1,SBNDDEE,0) = .45586E+02
xa1(0:7,TEV1,SBNDDEE,0) = (/ &
    .21084E+00, .99168E+00, .54407E+01,-.52851E+00, &
    .47493E+00,-.69595E+00, .12480E+02, .64027E+00 /)
xaillum(GEV350,TESLEE,0) = .15175E+02
xa1(0:7,GEV350,TESLEE,0) = (/ &
    .33093E+00, .11137E+01, .25275E+02,-.59942E+00, &
    .49623E+00,-.70403E+00, .60188E+02, .44637E+00 /)
xaillum(GEV500,TESLEE,0) = .21622E+02
xa1(0:7,GEV500,TESLEE,0) = (/ &
    .27175E+00, .10697E+01, .14858E+02,-.58418E+00, &
    .50824E+00,-.70387E+00, .36129E+02, .53002E+00 /)
xaillum(GEV800,TESLEE,0) = .43979E+02
xa1(0:7,GEV800,TESLEE,0) = (/ &
    .22994E+00, .10129E+01, .81905E+01,-.55751E+00, &
    .46551E+00,-.70461E+00, .19394E+02, .58387E+00 /)
xaillum(TEV1,TESLEE,0) = .25465E+02
xa1(0:7,TEV1,TESLEE,0) = (/ &
    .37294E+00, .67522E+00, .87504E+01,-.60576E+00, &
    .35095E+00,-.69821E+00, .18526E+02, .42784E+00 /)
xaillum(GEV500,XBNDDEE,0) = .13970E+02
xa1(0:7,GEV500,XBNDDEE,0) = (/ &
    .47296E+00, .46800E+00, .58897E+01,-.61689E+00, &
    .27181E+00,-.68923E+00, .10087E+02, .37462E+00 /)
xaillum(TEV1,XBNDDEE,0) = .41056E+02
xa1(0:7,TEV1,XBNDDEE,0) = (/ &
    .27965E+00, .74816E+00, .27415E+01,-.50491E+00, &
    .38320E+00,-.67945E+00, .47506E+01, .62218E+00 /)

```

Uses SBNDDEE 13a, TESLEE 13a, and XBNDDEE 13a.

Still unavailable

49b *⟨Initializations for circes 35b⟩+≡*

```

xaillum(GEV350,SBNDDEE,0) = -1d0
xaillum(GEV350,XBNDDEE,0) = -1d0
xaillum(GEV800,SBNDDEE,0) = -1d0
xaillum(GEV800,XBNDDEE,0) = -1d0

```

Uses SBNDDEE 13a and XBNDDEE 13a.

50a *<Initializations for circes 35b>+≡*

```

xaillum(GEV500,SBAND,0) = .31528E+02
xa1(0:7,GEV500,SBAND,0) = (/ &
    .38169E+00, .73949E+00, .12543E+02,-.61112E+00, &
    .51256E+00,-.69009E+00, .14892E+02, .63314E+00 /)
xaillum(TEV1,SBAND,0) = .24613E+03
xa1(0:7,TEV1,SBAND,0) = (/ &
    .24256E+00, .94117E+00, .66775E+01,-.55160E+00, &
    .57484E+00,-.68891E+00, .92271E+01, .81162E+00 /)
xaillum(GEV350,TESLA,0) = .74549E+02
xa1(0:7,GEV350,TESLA,0) = (/ &
    .34120E+00, .12230E+01, .32932E+02,-.59850E+00, &
    .65947E+00,-.69574E+00, .38116E+02, .63879E+00 /)
xaillum(GEV500,TESLA,0) = .10668E+03
xa1(0:7,GEV500,TESLA,0) = (/ &
    .28082E+00, .11074E+01, .18399E+02,-.59118E+00, &
    .68880E+00,-.69375E+00, .23463E+02, .76073E+00 /)
xaillum(GEV800,TESLA,0) = .29006E+03
xa1(0:7,GEV800,TESLA,0) = (/ &
    .21272E+00, .11443E+01, .92564E+01,-.54657E+00, &
    .66799E+00,-.69137E+00, .12498E+02, .87571E+00 /)
xaillum(TEV1,TESLA,0) = .11009E+03
xa1(0:7,TEV1,TESLA,0) = (/ &
    .41058E+00, .64745E+00, .11271E+02,-.61996E+00, &
    .39801E+00,-.69150E+00, .14560E+02, .49924E+00 /)
xaillum(GEV500,XBAND,0) = .36179E+02
xa1(0:7,GEV500,XBAND,0) = (/ &
    .51155E+00, .43313E+00, .70446E+01,-.63003E+00, &
    .29449E+00,-.68747E+00, .83489E+01, .42458E+00 /)
xaillum(TEV1,XBAND,0) = .11748E+03
xa1(0:7,TEV1,XBAND,0) = (/ &
    .32917E+00, .54322E+00, .28493E+01,-.57959E+00, &
    .39266E+00,-.68217E+00, .38475E+01, .68478E+00 /)

```

Uses SBAND 13a, TESLA 13a, and XBAND 13a.

Still unavailable

50b *<Initializations for circes 35b>+≡*

```

xaillum(GEV350,SBAND,0) = -1d0
xaillum(GEV350,XBAND,0) = -1d0
xaillum(GEV800,SBAND,0) = -1d0
xaillum(GEV800,XBAND,0) = -1d0

```

Uses SBAND 13a and XBAND 13a.

6.2.2 Version 2

50c *<Version 2 has been retired 50c>≡*

```

call circem ('PANIC', '*****')
call circem ('PANIC', '* version 2 has been retired, *')
call circem ('PANIC', '* please use version 1 instead! *')
call circem ('PANIC', '*****')

```

```

        return
    Uses circem 87a.

```

6.2.3 Versions 3 and 4

- 51a *<Update version 3 and 4 derived parameters in circe1 parameters 51a>*≡
- ```

 if (circe1_params%rev .eq. 0) then
 r = 0
 elseif (circe1_params%rev .ge. 19970417) then
 r = 5
 if (ver34 .eq. 3) then
 call circem ('WARNING', 'version 3 retired after 97/04/17')
 call circem ('MESSAGE', 'falling back to version 4')
 end if
 else if (circe1_params%rev .ge. 19961022) then
 r = ver34
 if ((circe1_params%roots .ne. 800d0) .or. (circe1_params%acc .ne. TESLA)) then
 call circem ('ERROR', 'versions 3 and 4 before 97/04/17')
 call circem ('ERROR', 'apply to TESLA at 800 GeV only')
 call circem ('MESSAGE', 'falling back to TESLA at 800GeV')
 circe1_params%acc = TESLA
 e = GEV800
 end if
 else if (circe1_params%rev .lt. 19961022) then
 call circem ('ERROR', &
 'no revision of versions 3 and 4 available before 96/10/22')
 call circem ('MESSAGE', 'falling back to default')
 r = 5
 end if
 <Log revision mapping 38a>
 Uses TESLA 13a and circem 87a.

```
- 51b *<Update version 3 and 4 derived parameters in circe1 parameters 51a>*+≡
- <Map roots to e 38c>*
- ```

    if (xa3lum(e,circe1_params%acc,r) .lt. 0d0) then
        write (msgbuf, 2002) circe1_params%roots, accnam(circe1_params%acc), r
        call circem ('ERROR', msgbuf)
        call circem ('MESSAGE', 'falling back to 500GeV')
        e = GEV500
    endif
    <Log energy mapping 39d>
    Uses circem 87a.

```
- 51c *<Local variables for circes 33c>*+≡
- ```

 integer, parameter :: A3NEGY = 5, A3NREV = 5

```
- 51d *<Update version 3 and 4 derived parameters in circe1 parameters 51a>*+≡
- ```

    circe1_params%lumi = xa3lum (e,circe1_params%acc,r)
    do i = 0, 7
        circe1_params%a1(i) = xa3(i,e,circe1_params%acc,r)
    end do

```

52a *<Local variables for circes 33c>+≡*
`real, dimension(A3NEGY,NACC,0:A3NREV), save :: xa3lum = -1`
`real, dimension(0:7,A3NEGY,NACC,0:A3NREV), save :: xa3 = 0`
 Uses NACC 13b.

Revisions 3 & 4. The mother of all revisions.

52b *<Initializations for circes 35b>+≡*
`xa3lum(GEV800,TESLA,3) = .17196E+03`
`xa3(0:7,GEV800,TESLA,3) = (/ &`
`.21633E+00, .11333E+01, .95928E+01,-.55095E+00, &`
`.73044E+00,-.69101E+00, .12868E+02, .94737E+00 /)`
`xa3lum(GEV800,TESLA, 4) = .16408E+03`
`xa3(0:7,GEV800,TESLA, 4) = (/ &`
`.41828E+00, .72418E+00, .14137E+02,-.61189E+00, &`
`.36697E+00,-.69205E+00, .17713E+02, .43583E+00 /)`

Uses TESLA 13a.

Revision 5.

52c *<Initializations for circes 35b>+≡*
`xa3lum(GEV350,TESLA,5) = 0.66447E+02`
`xa3(0:7,GEV350,TESLA,5) = (/ &`
`0.69418E+00, 0.50553E+00, 0.48430E+02,-0.63911E+00, &`
`0.34074E+00,-0.69533E+00, 0.55502E+02, 0.29397E+00 /)`
`xa3lum(GEV500,TESLA,5) = 0.95241E+02`
`xa3(0:7,GEV500,TESLA,5) = (/ &`
`0.64882E+00, 0.45462E+00, 0.27103E+02,-0.64535E+00, &`
`0.35101E+00,-0.69467E+00, 0.33658E+02, 0.35024E+00 /)`
`xa3lum(GEV800,TESLA,5) = 0.16974E+03`
`xa3(0:7,GEV800,TESLA,5) = (/ &`
`0.58706E+00, 0.43771E+00, 0.13422E+02,-0.63804E+00, &`
`0.35541E+00,-0.69467E+00, 0.17528E+02, 0.43051E+00 /)`
`xa3lum(TEV1,TESLA,5) = 0.21222E+03`
`xa3(0:7,TEV1,TESLA,5) = (/ &`
`0.55525E+00, 0.42577E+00, 0.96341E+01,-0.63587E+00, &`
`0.36448E+00,-0.69365E+00, 0.13161E+02, 0.47715E+00 /)`
`xa3lum(TEV16,TESLA,5) = 0.34086E+03`
`xa3(0:7,TEV16,TESLA,5) = (/ &`
`0.49058E+00, 0.42609E+00, 0.50550E+01,-0.61867E+00, &`
`0.39225E+00,-0.68916E+00, 0.75514E+01, 0.58754E+00 /)`

Uses TESLA 13a.

Revision 0. Currently identical to revision 5.

52d *<Initializations for circes 35b>+≡*
`xa3lum(GEV350,TESLA,0) = 0.66447E+02`
`xa3(0:7,GEV350,TESLA,0) = (/ &`
`0.69418E+00, 0.50553E+00, 0.48430E+02,-0.63911E+00, &`
`0.34074E+00,-0.69533E+00, 0.55502E+02, 0.29397E+00 /)`
`xa3lum(GEV500,TESLA,0) = 0.95241E+02`
`xa3(0:7,GEV500,TESLA,0) = (/ &`
`0.64882E+00, 0.45462E+00, 0.27103E+02,-0.64535E+00, &`
`0.35101E+00,-0.69467E+00, 0.33658E+02, 0.35024E+00 /)`

```

xa3lum(GEV800,TESLA,0) = 0.16974E+03
xa3(0:7,GEV800,TESLA,0) = (/ &
    0.58706E+00, 0.43771E+00, 0.13422E+02,-0.63804E+00, &
    0.35541E+00,-0.69467E+00, 0.17528E+02, 0.43051E+00 /)
xa3lum(TEV1,TESLA,0) = 0.21222E+03
xa3(0:7,TEV1,TESLA,0) = (/ &
    0.55525E+00, 0.42577E+00, 0.96341E+01,-0.63587E+00, &
    0.36448E+00,-0.69365E+00, 0.13161E+02, 0.47715E+00 /)
xa3lum(TEV16,TESLA,0) = 0.34086E+03
xa3(0:7,TEV16,TESLA,0) = (/ &
    0.49058E+00, 0.42609E+00, 0.50550E+01,-0.61867E+00, &
    0.39225E+00,-0.68916E+00, 0.75514E+01, 0.58754E+00 /)

```

Uses TESLA 13a.

6.2.4 Version 5

53a *<Update version 5 derived parameters in circe1 parameters 53a>≡*

```

    if (circe1_params%rev .eq. 0) then
        r = 0
    elseif (circe1_params%rev .ge. 19980505) then
        r = 1
    elseif (circe1_params%rev .lt. 19980505) then
        call circem ('ERROR', &
            'no revision of version 5 available before 98/05/05')
        call circem ('MESSAGE', 'falling back to default')
        r = 1
    endif
    <Log revision mapping 38a>

```

Uses circem 87a.

53b *<Update version 5 derived parameters in circe1 parameters 53a>+≡*

```

    if (circe1_params%acc .ne. TESLA) then
        call circem ('ERROR', 'versions 5 applies to TESLA only')
        circe1_params%acc = TESLA
    end if

```

<Map roots to e 38c>

```

    if (xa5lum(e,circe1_params%acc,r) .lt. 0d0) then
        write (msgbuf, 2002) circe1_params%roots, accnam(circe1_params%acc), r
        call circem ('ERROR', msgbuf)
        call circem ('MESSAGE', 'falling back to 500GeV')
        e = GEV500
    endif
    <Log energy mapping 39d>

```

Uses TESLA 13a and circem 87a.

53c *<Local variables for circes 33c>+≡*

```

    integer, parameter :: A5NEGY = 5, A5NREV = 1

```

53d *<Update version 5 derived parameters in circe1 parameters 53a>+≡*

```

    circe1_params%lumi = xa5lum (e,circe1_params%acc,r)
    do i = 0, 7

```

```

        circe1_params%a1(i) = xa5(i,e,circe1_params%acc,r)
    end do

```

54a *<Local variables for circes 33c>+≡*
 real, dimension(A5NEGY,NACC,0:A5NREV), save :: xa5lum
 real, dimension(0:7,A5NEGY,NACC,0:A5NREV), save :: xa5
 Uses NACC 13b.

Revision 1. The mother of all revisions. Note that $3.3980 \cdot 10^{34} \text{ cm}^{-2} \text{ s}^{-1} = 2.4099 \cdot 10^{34} \text{ m}^{-2} \cdot 2820.5 \text{ s}^{-1}$ and $3.5936 \cdot 10^{34} \text{ cm}^{-2} \text{ s}^{-1} = 2.6619 \cdot 10^{34} \text{ m}^{-2} \cdot 4500.3 \text{ s}^{-1}$. This unit conversion is missing in *all* earlier versions, unfortunately.

54b *<Initializations for circes 35b>+≡*
 xa5lum(GEV350,TESLA,1) = -1.0
 xa5lum(GEV500,TESLA,1) = 0.33980E+03
 xa5(0:7,GEV500,TESLA,1) = (/ &
 0.49808E+00, 0.54613E+00, 0.12287E+02,-0.62756E+00, &
 0.42817E+00,-0.69120E+00, 0.17067E+02, 0.51143E+00 /)
 xa5lum(GEV800,TESLA,1) = 0.35936E+03
 xa5(0:7,GEV800,TESLA,1) = (/ &
 0.58751E+00, 0.43128E+00, 0.13324E+02,-0.64006E+00, &
 0.30682E+00,-0.69235E+00, 0.16815E+02, 0.37078E+00 /)
 xa5lum(TEV1, TESLA,1) = -1.0
 xa5lum(TEV16,TESLA,1) = -1.0

Uses TESLA 13a.

Revision 0. Currently identical to revision 1.

54c *<Initializations for circes 35b>+≡*
 xa5lum(GEV350,TESLA,0) = -1.0
 xa5lum(GEV500,TESLA,0) = 0.33980E+03
 xa5(0:7,GEV500,TESLA,0) = (/ &
 0.49808E+00, 0.54613E+00, 0.12287E+02,-0.62756E+00, &
 0.42817E+00,-0.69120E+00, 0.17067E+02, 0.51143E+00 /)
 xa5lum(GEV800,TESLA,0) = 0.35936E+03
 xa5(0:7,GEV800,TESLA,0) = (/ &
 0.58751E+00, 0.43128E+00, 0.13324E+02,-0.64006E+00, &
 0.30682E+00,-0.69235E+00, 0.16815E+02, 0.37078E+00 /)
 xa5lum(TEV1, TESLA,0) = -1.0
 xa5lum(TEV16,TESLA,0) = -1.0

Uses TESLA 13a.

6.2.5 Version 6

54d *<Update version 6 derived parameters in circe1 parameters 54d>≡*
 if (circe1_params%rev .eq. 0) then
 r = 0
 else if (circe1_params%rev .ge. 19990415) then
 r = 1
 else if (circe1_params%rev .lt. 19990415) then
 call circem ('ERROR', &
 'no revision of version 6 available before 1999/04/15')
 call circem ('MESSAGE', 'falling back to default')

```

        r = 1
    end if
    <Log revision mapping 38a>
    Uses circem 87a.
55a <Update version 6 derived parameters in circe1 parameters 54d>+=
    if (circe1_params%acc .ne. TESLA) then
        call circem ('ERROR', 'versions 6 applies to TESLA only')
        circe1_params%acc = TESLA
    end if
    <Map roots to e at low energies 55b>
    if (xa6lum(e,circe1_params%acc,r) .lt. 0d0) then
        write (msgbuf, 2002) circe1_params%roots, accnam(circe1_params%acc), r
        call circem ('ERROR', msgbuf)
        call circem ('MESSAGE', 'falling back to 500GeV')
        e = GEV500
    endif
    <Log energy mapping 39d>
    Uses TESLA 13a and circem 87a.
55b <Map roots to e at low energies 55b>≡
    if (circe1_params%roots .eq. 90d0) then
        e = GEV090
    elseif ((circe1_params%roots .ge. 85d0) .and. (circe1_params%roots .le. 95d0)) then
        write (msgbuf, 2001) circe1_params%roots, 90d0
        call circem ('MESSAGE', msgbuf)
        e = GEV090
    elseif (circe1_params%roots .eq. 170d0) then
        e = GEV170
    elseif ((circe1_params%roots .ge. 160d0) .and. (circe1_params%roots .le. 180d0)) then
        write (msgbuf, 2001) circe1_params%roots, 170d0
        call circem ('MESSAGE', msgbuf)
        e = GEV170
    elseif (circe1_params%roots .eq. 350d0) then
        e = GEV350
    elseif ((circe1_params%roots .ge. 340d0) .and. (circe1_params%roots .le. 370d0)) then
        write (msgbuf, 2001) circe1_params%roots, 350d0
        call circem ('MESSAGE', msgbuf)
        e = GEV350
    elseif (circe1_params%roots .eq. 500d0) then
        e = GEV500
    elseif ((circe1_params%roots .ge. 480d0) .and. (circe1_params%roots .le. 520d0)) then
        write (msgbuf, 2001) circe1_params%roots, 500d0
        call circem ('MESSAGE', msgbuf)
        e = GEV500
    else
        call circem ('ERROR', &
            'only ROOTS = 90, 170, 350, and 500GeV available')
        call circem ('MESSAGE', 'falling back to 500GeV')
        e = GEV500
    endif

```

Uses [circem 87a](#).

```
56a <Local variables for circes 33c>+≡
      integer, parameter :: A6NEGY = 2, A6NREV = 1

56b <Update version 6 derived parameters in circe1 parameters 54d>+≡
      circe1_params%lumi = xa6lum (e,circe1_params%acc,r)
      do i = 0, 7
        circe1_params%a1(i) = xa6(i,e,circe1_params%acc,r)
      end do

56c <Local variables for circes 33c>+≡
      real, dimension(GEV090:A6NEGY,NACC,0:A6NREV), save :: xa6lum
      real, dimension(0:7,GEV090:A6NEGY,NACC,0:A6NREV), save :: xa6
```

Uses [NACC 13b](#).

Revision 1. The mother of all revisions.

```
56d <Initializations for circes 35b>+≡
      xa6lum(GEV090,TESLA,1) = 0.62408E+02
      xa6(0:7,GEV090,TESLA,1) = (/ &
        0.72637E+00, 0.75534E+00, 0.18180E+03,-0.63426E+00, &
        0.36829E+00,-0.69653E+00, 0.18908E+03, 0.22157E+00 /)
      xa6lum(GEV170,TESLA,1) = 0.11532E+02
      xa6(0:7,GEV170,TESLA,1) = (/ &
        0.65232E+00, 0.67249E+00, 0.66862E+02,-0.63315E+00, &
        0.38470E+00,-0.69477E+00, 0.75120E+02, 0.30162E+00 /)
      xa6lum(GEV350,TESLA,1) = 0.24641E+03
      xa6(0:7,GEV350,TESLA,1) = (/ &
        0.54610E+00, 0.59105E+00, 0.20297E+02,-0.62747E+00, &
        0.41588E+00,-0.69188E+00, 0.26345E+02, 0.43818E+00 /)
      xa6lum(GEV500,TESLA,1) = 0.30340E+03
      xa6(0:7,GEV500,TESLA,1) = (/ &
        0.52744E+00, 0.52573E+00, 0.13895E+02,-0.63145E+00, &
        0.40824E+00,-0.69150E+00, 0.18645E+02, 0.47585E+00 /)
```

Uses [TESLA 13a](#).

Revision 0. Currently identical to revision 1.

```
56e <Initializations for circes 35b>+≡
      xa6lum(GEV090,TESLA,0) = 0.62408E+02
      xa6(0:7,GEV090,TESLA,0) = (/ &
        0.72637E+00, 0.75534E+00, 0.18180E+03,-0.63426E+00, &
        0.36829E+00,-0.69653E+00, 0.18908E+03, 0.22157E+00 /)
      xa6lum(GEV170,TESLA,0) = 0.11532E+02
      xa6(0:7,GEV170,TESLA,0) = (/ &
        0.65232E+00, 0.67249E+00, 0.66862E+02,-0.63315E+00, &
        0.38470E+00,-0.69477E+00, 0.75120E+02, 0.30162E+00 /)
      xa6lum(GEV350,TESLA,0) = 0.24641E+03
      xa6(0:7,GEV350,TESLA,0) = (/ &
        0.54610E+00, 0.59105E+00, 0.20297E+02,-0.62747E+00, &
        0.41588E+00,-0.69188E+00, 0.26345E+02, 0.43818E+00 /)
      xa6lum(GEV500,TESLA,0) = 0.30340E+03
      xa6(0:7,GEV500,TESLA,0) = (/ &
```

```

0.52744E+00, 0.52573E+00, 0.13895E+02,-0.63145E+00, &
0.40824E+00,-0.69150E+00, 0.18645E+02, 0.47585E+00 /)

```

Uses TESLA 13a.

6.2.6 Version 7

57a *<Update version 7 derived parameters in circe1 parameters 57a>*≡

```

if (circe1_params%rev .eq. 0) then
  r = 0
elseif (circe1_params%rev .ge. 20000426) then
  r = 1
elseif (circe1_params%rev .lt. 20000426) then
  call circem ('ERROR', &
    'no revision of version 7 available before 2000/04/26')
  call circem ('MESSAGE', 'falling back to default')
  r = 1
endif

```

<Log revision mapping 38a>

Uses circem 87a.

57b *<Update version 7 derived parameters in circe1 parameters 57a>*+≡

```

if (circe1_params%acc .ne. TESLA .and. circe1_params%acc .ne. JLCNLC) then
  call circem ('ERROR', &
    'version 7 applies to TESLA and JLCNLC only')
  call circem ('ERROR', 'falling back to TESLA')
  circe1_params%acc = TESLA
endif

```

<Linearly interpolate energies 57e>

<Log energy mapping 39d>

Uses JLCNLC 13a, TESLA 13a, and circem 87a.

57c *<formats for circles 38d>*+≡

```

2004 format ('energy ', F6.1, 'GeV too low, using spectrum for ', F6.1, 'GeV')
2005 format ('energy ', F6.1, 'GeV too high, using spectrum for ', F6.1, 'GeV')
2006 format ('energy ', F6.1, 'GeV interpolated between ', F6.1, ' and ', F6.1, 'GeV')

```

57d *<Local variables for circles 33c>*+≡

```

real(kind=double) :: eloal, ehival
real(kind=double), parameter :: DELTAE = 0.5d0

```

The rules are as follows: XBAND has 500 GeV and 1 TeV, TESLA has 500 GeV and 800 TeV. Low energy TESLA will be added.

57e *<Linearly interpolate energies 57e>*≡

```

e = GEV090 - 1
elo = e
ehi = e
if (circe1_params%acc .eq. TESLA) then
  if (circe1_params%roots .lt. 90d0 - DELTAE) then
    write (msgbuf, 2004) circe1_params%roots, 90d0
    call circem ('MESSAGE', msgbuf)
    e = GEV090
  elseif (abs (circe1_params%roots-090d0) .le. DELTAE) then

```

```

        e = GEV090
    elseif (circe1_params%roots .lt. 170d0 - DELTAE) then
        write (msgbuf, 2005) circe1_params%roots, 170d0
        call circem ('MESSAGE', msgbuf)
        e = GEV170
    elseif (abs (circe1_params%roots-170d0) .le. DELTAE) then
        e = GEV170
    elseif (circe1_params%roots .lt. 350d0-DELTAE) then
        write (msgbuf, 2006) circe1_params%roots, 170d0, 350d0
        call circem ('MESSAGE', msgbuf)
        elo = GEV170
        ehi = GEV350
        eloal = 170d0
        ehival = 350d0
    elseif (abs (circe1_params%roots-350d0) .le. DELTAE) then
        e = GEV350
    elseif (circe1_params%roots .lt. 500d0 - DELTAE) then
        write (msgbuf, 2006) circe1_params%roots, 350d0, 500d0
        call circem ('MESSAGE', msgbuf)
        elo = GEV350
        ehi = GEV500
        eloal = 350d0
        ehival = 500d0
    elseif (abs (circe1_params%roots-500d0) .le. DELTAE) then
        e = GEV500
    elseif (circe1_params%roots .lt. 800d0 - DELTAE) then
        write (msgbuf, 2006) circe1_params%roots, 500d0, 800d0
        call circem ('MESSAGE', msgbuf)
        elo = GEV500
        ehi = GEV800
        eloal = 500d0
        ehival = 800d0
    elseif (abs (circe1_params%roots-800d0) .le. DELTAE) then
        e = GEV800
    else
        write (msgbuf, 2005) circe1_params%roots, 800d0
        call circem ('MESSAGE', msgbuf)
        e = GEV800
    endif
elseif (circe1_params%acc .eq. XBAND) then
    if (circe1_params%roots .lt. 500d0 - DELTAE) then
        write (msgbuf, 2004) circe1_params%roots, 500d0
        call circem ('MESSAGE', msgbuf)
        e = GEV500
    elseif (abs (circe1_params%roots-500d0) .le. DELTAE) then
        e = GEV500
    elseif (circe1_params%roots .lt. 1000d0 - DELTAE) then
        write (msgbuf, 2006) circe1_params%roots, 500d0, 1000d0
        call circem ('MESSAGE', msgbuf)
        elo = GEV500

```

```

        ehi = TEV1
        eloal = 500d0
        ehival = 1000d0
    elseif (abs (circe1_params%roots-1000d0) .le. DELTAE) then
        e = TEV1
    else
        write (msgbuf, 2005) circe1_params%roots, 1000d0
        call circem ('MESSAGE', msgbuf)
        e = TEV1
    endif
endif

```

Uses TESLA 13a, XBAND 13a, and circem 87a.

59a *<Local variables for circes 33c>+≡*
integer, parameter :: A7NEGY = TEV1, A7NREV = 1

Note that ew *must not* interpolate a1(0) and a1(7) because they depend non-linearly on the other parameters!

59b *<Update version 7 derived parameters in circe1 parameters 57a>+≡*
if (e .ge. GEV090) then
 circe1_params%lumi = xa7lum(e,circe1_params%acc,r)
 do i = 0, 7
 circe1_params%a1(i) = xa7(i,e,circe1_params%acc,r)
 end do
else if (elo .ge. GEV090 .and. ehi .ge. GEV090) then
 circe1_params%lumi = ((circe1_params%roots-eloal)*xa7lum(ehi,circe1_params%acc,
 + (ehival-circe1_params%roots)*xa7lum(elo,circe1_params%acc,r)) / (ehival -
 do i = 1, 6
 circe1_params%a1(i) = ((circe1_params%roots-eloal)*xa7(i,ehi,circe1_params%
 + (ehival-circe1_params%roots)*xa7(i,elo,circe1_params%acc,r)) / (ehival -
 end do
 circe1_params%a1(0) = 1d0 - circe1_params%a1(1) * beta(circe1_params%a1(2)+1d0,
 circe1_params%a1(7) = circe1_params%a1(4) * beta(circe1_params%a1(5)+1d0,circe1
endif

Uses beta 105.

59c *<Local variables for circes 33c>+≡*
real, dimension(GEV090:A7NEGY,NACC,0:A7NREV), save :: xa7lum
real, dimension(0:7,GEV090:A7NEGY,NACC,0:A7NREV), save :: xa7

Uses NACC 13b.

Revision 1. The mother of all revisions.

59d *<Initializations for circes 35b>+≡*
xa7lum(GEV090,TESLA,1) = 0.62408E+02
xa7(0:7,GEV090,TESLA,1) = (/ &
 0.72637E+00, 0.75534E+00, 0.18180E+03,-0.63426E+00, &
 0.36829E+00,-0.69653E+00, 0.18908E+03, 0.22157E+00 /)
xa7lum(GEV170,TESLA,1) = 0.11532E+02
xa7(0:7,GEV170,TESLA,1) = (/ &
 0.65232E+00, 0.67249E+00, 0.66862E+02,-0.63315E+00, &
 0.38470E+00,-0.69477E+00, 0.75120E+02, 0.30162E+00 /)
xa7lum(GEV350,TESLA,1) = 0.24641E+03

```

xa7(0:7,GEV350,TESLA,1) = (/ &
    0.54610E+00, 0.59105E+00, 0.20297E+02,-0.62747E+00, &
    0.41588E+00,-0.69188E+00, 0.26345E+02, 0.43818E+00 /)
xa7lum(GEV500,TESLA,1) = 0.34704E+03
xa7(0:7,GEV500,TESLA,1) = (/ &
    0.51288E+00, 0.49025E+00, 0.99716E+01,-0.62850E+00, &
    0.41048E+00,-0.69065E+00, 0.13922E+02, 0.51902E+00 /)
xa7lum(GEV800,TESLA,1) = 0.57719E+03
xa7(0:7,GEV800,TESLA,1) = (/ &
    0.52490E+00, 0.42573E+00, 0.69069E+01,-0.62649E+00, &
    0.32380E+00,-0.68958E+00, 0.93819E+01, 0.45671E+00 /)
xa7lum(TEV1,TESLA,1) = -1.0

```

Uses TESLA 13a.

60a *<Initializations for circes 35b>+≡*

```

xa7lum(GEV090,JLCNLC,1) = -1.0
xa7lum(GEV170,JLCNLC,1) = -1.0
xa7lum(GEV350,JLCNLC,1) = -1.0
xa7lum(GEV500,JLCNLC,1) = 0.63039E+02
xa7(0:7,GEV500,JLCNLC,1) = (/ &
    0.58967E+00, 0.34035E+00, 0.63631E+01,-0.63683E+00, &
    0.33383E+00,-0.68803E+00, 0.81005E+01, 0.48702E+00 /)
xa7lum(TEV1,JLCNLC,1) = 0.12812E+03
xa7(0:7,TEV1,JLCNLC,1) = (/ &
    0.50222E+00, 0.33773E+00, 0.25681E+01,-0.61711E+00, &
    0.36826E+00,-0.68335E+00, 0.36746E+01, 0.65393E+00 /)

```

Uses JLCNLC 13a.

Revision 0.

60b *<Initializations for circes 35b>+≡*

```

xa7lum(GEV090,TESLA,0) = 0.62408E+02
xa7(0:7,GEV090,TESLA,0) = (/ &
    0.72637E+00, 0.75534E+00, 0.18180E+03,-0.63426E+00, &
    0.36829E+00,-0.69653E+00, 0.18908E+03, 0.22157E+00 /)
xa7lum(GEV170,TESLA,0) = 0.11532E+02
xa7(0:7,GEV170,TESLA,0) = (/ &
    0.65232E+00, 0.67249E+00, 0.66862E+02,-0.63315E+00, &
    0.38470E+00,-0.69477E+00, 0.75120E+02, 0.30162E+00 /)
xa7lum(GEV350,TESLA,0) = 0.24641E+03
xa7(0:7,GEV350,TESLA,0) = (/ &
    0.54610E+00, 0.59105E+00, 0.20297E+02,-0.62747E+00, &
    0.41588E+00,-0.69188E+00, 0.26345E+02, 0.43818E+00 /)
xa7lum(GEV500,TESLA,0) = 0.34704E+03
xa7(0:7,GEV500,TESLA,0) = (/ &
    0.51288E+00, 0.49025E+00, 0.99716E+01,-0.62850E+00, &
    0.41048E+00,-0.69065E+00, 0.13922E+02, 0.51902E+00 /)
xa7lum(GEV800,TESLA,0) = 0.57719E+03
xa7(0:7,GEV800,TESLA,0) = (/ &
    0.52490E+00, 0.42573E+00, 0.69069E+01,-0.62649E+00, &
    0.32380E+00,-0.68958E+00, 0.93819E+01, 0.45671E+00 /)
xa7lum(TEV1,TESLA,0) = -1.0

```

Uses TESLA 13a.

```
61a <Initializations for circes 35b>+≡
    xa7lum(GEV090,JLCNLC,0) = -1.0
    xa7lum(GEV170,JLCNLC,0) = -1.0
    xa7lum(GEV350,JLCNLC,0) = -1.0
    xa7lum(GEV500,JLCNLC,0) = 0.63039E+02
    xa7(0:7,GEV500,JLCNLC,0) = (/ &
        0.58967E+00, 0.34035E+00, 0.63631E+01,-0.63683E+00, &
        0.33383E+00,-0.68803E+00, 0.81005E+01, 0.48702E+00 /)
    xa7lum(TEV1,JLCNLC,0) = 0.12812E+03
    xa7(0:7,TEV1,JLCNLC,0) = (/ &
        0.50222E+00, 0.33773E+00, 0.25681E+01,-0.61711E+00, &
        0.36826E+00,-0.68335E+00, 0.36746E+01, 0.65393E+00 /)
```

Uses JLCNLC 13a.

6.2.7 Version 8

```
61b <Update version 8 derived parameters in circe1 parameters 61b>≡
    if (circe1_params%rev .eq. 0) then
        r = 0
    elseif (circe1_params%rev .ge. 20010617) then
        r = 1
    elseif (circe1_params%rev .lt. 20010617) then
        call circem ('ERROR', &
            'no revision of version 8 available before 2001/06/17')
        call circem ('MESSAGE', 'falling back to default')
        r = 1
    endif
    <Log revision mapping 38a>
```

Uses circem 87a.

```
61c <Update version 8 derived parameters in circe1 parameters 61b>+≡
    if (circe1_params%acc .eq. NLCH) then
        circe1_params%acc = JLCNLC
    end if
    if (circe1_params%acc .ne. JLCNLC) then
        call circem ('ERROR', &
            'version 8 applies to JLCNLC (NLC H) only')
        call circem ('ERROR', 'falling back to JLCNLC')
        circe1_params%acc = JLCNLC
    end if
    <Linearly interpolate energies 57e>
    <Log energy mapping 39d>
```

Uses JLCNLC 13a and circem 87a.

```
61d <Local variables for circes 33c>+≡
```

```
    integer, parameter :: A8NEGY = TEV1, A8NREV = 1
```

Note that ew *must not* interpolate a1(0) and a1(7) because they depend non-linearly on the other parameters!

```
61e <Update version 8 derived parameters in circe1 parameters 61b>+≡
```

```

if (e .ge. GEV090) then
  circe1_params%lumi = xa8lum(e,circe1_params%acc,r)
  do i = 0, 7
    circe1_params%a1(i) = xa8(i,e,circe1_params%acc,r)
  end do
elseif (elo .ge. GEV090 .and. ehi .ge. GEV090) then
  circe1_params%lumi = ((circe1_params%roots-eloval)*xa8lum(ehi,circe1_params%acc,r)
    + (ehival-circe1_params%roots)*xa8lum(elo,circe1_params%acc,r)) / (ehival - elo)
  do i = 1, 6
    circe1_params%a1(i) = ((circe1_params%roots-eloval)*xa8(i,ehi,circe1_params%acc,r)
      + (ehival-circe1_params%roots)*xa8(i,elo,circe1_params%acc,r)) / (ehival - elo)
  end do
  circe1_params%a1(0) = 1d0 - circe1_params%a1(1) * beta(circe1_params%a1(2)+1d0,
    circe1_params%a1(7) = circe1_params%a1(4) * beta(circe1_params%a1(5)+1d0,circe1_params%a1(7))
endif

```

Uses beta 105.

62a *<Local variables for circes 33c>+≡*

```

real, dimension(GEV090:A8NEGY,NACC,0:A8NREV), save :: xa8lum
real, dimension(0:7,GEV090:A8NEGY,NACC,0:A8NREV), save :: xa8

```

Uses NACC 13b.

Revision 1. The mother of all revisions.

62b *<Initializations for circes 35b>+≡*

```

xa8lum(GEV090,TESLA,1) = -1.0
xa8lum(GEV170,TESLA,1) = -1.0
xa8lum(GEV350,TESLA,1) = -1.0
xa8lum(GEV500,TESLA,1) = -1.0
xa8lum(GEV800,TESLA,1) = -1.0
xa8lum(TEV1, TESLA,1) = -1.0

```

Uses TESLA 13a.

62c *<Initializations for circes 35b>+≡*

```

xa8lum(GEV090,JLCNLC,1) = -1.0
xa8lum(GEV170,JLCNLC,1) = -1.0
xa8lum(GEV350,JLCNLC,1) = -1.0
xa8lum(GEV500,JLCNLC,1) = 0.239924E+03
xa8(0:7,GEV500,JLCNLC,1) = (/ &
  0.57025E+00, 0.34004E+00, 0.52864E+01,-0.63405E+00, &
  0.31627E+00,-0.68722E+00, 0.69629E+01, 0.47973E+00 /)
xa8lum(TEV1,JLCNLC,1) = 0.40858E+03
xa8(0:7,TEV1,JLCNLC,1) = (/ &
  0.52344E+00, 0.31536E+00, 0.25244E+01,-0.62215E+00, &
  0.31935E+00,-0.68424E+00, 0.35877E+01, 0.57315E+00 /)

```

Uses JLCNLC 13a.

Revision 0.

62d *<Initializations for circes 35b>+≡*

```

xa8lum(GEV090,TESLA,0) = -1.0
xa8lum(GEV170,TESLA,0) = -1.0
xa8lum(GEV350,TESLA,0) = -1.0
xa8lum(GEV500,TESLA,0) = -1.0

```

```

xa8lum(GEV800, TESLA, 0) = -1.0
xa8lum(TEV1, TESLA, 0) = -1.0

```

Uses TESLA 13a.

```

63a <Initializations for circes 35b>+≡
    xa8lum(GEV090, JLCNLC, 0) = -1.0
    xa8lum(GEV170, JLCNLC, 0) = -1.0
    xa8lum(GEV350, JLCNLC, 0) = -1.0
    xa8lum(GEV500, JLCNLC, 0) = 0.239924E+03
    xa8(0:7, GEV500, JLCNLC, 0) = (/ &
        0.57025E+00, 0.34004E+00, 0.52864E+01, -0.63405E+00, &
        0.31627E+00, -0.68722E+00, 0.69629E+01, 0.47973E+00 /)
    xa8lum(TEV1, JLCNLC, 0) = 0.40858E+03
    xa8(0:7, TEV1, JLCNLC, 0) = (/ &
        0.52344E+00, 0.31536E+00, 0.25244E+01, -0.62215E+00, &
        0.31935E+00, -0.68424E+00, 0.35877E+01, 0.57315E+00 /)

```

Uses JLCNLC 13a.

6.2.8 Version 9

```

63b <Update version 9 derived parameters in circe1 parameters 63b>≡
    if (circe1_params%rev .eq. 0) then
        r = 0
    elseif (circe1_params%rev .ge. 20020328) then
        r = 1
    elseif (circe1_params%rev .lt. 20020328) then
        call circem ('ERROR', &
            'no revision of version 9 available before 2002/03/28')
        call circem ('MESSAGE', 'falling back to default')
        r = 1
    endif
    <Log revision mapping 38a>

```

Uses circem 87a.

```

63c <Update version 9 derived parameters in circe1 parameters 63b>+≡
    if (circe1_params%acc .ne. JLCNLC .and. circe1_params%acc .ne. NLCH) then
        call circem ('ERROR', &
            'version 9 applies to JLCNLC and NLCH only')
        call circem ('ERROR', 'falling back to JLCNLC')
        circe1_params%acc = JLCNLC
    end if
    if (circe1_params%acc .eq. JLCNLC) then
        <Linearly interpolate energies for JLC/NLC 2002 63d>
    else if (circe1_params%acc .eq. NLCH) then
        <Linearly interpolate energies for NLC H 2002 65a>
    end if
    <Log energy mapping 39d>

```

Uses JLCNLC 13a and circem 87a.

```

63d <Linearly interpolate energies for JLC/NLC 2002 63d>≡
    e = GEV090 - 1

```

```

elo = e
ehi = e
if (circe1_params%roots .lt. 250d0 - DELTAE) then
    write (msgbuf, 2004) circe1_params%roots, 250d0
    call circem ('MESSAGE', msgbuf)
    e = GEV250
elseif (abs (circe1_params%roots-250d0) .le. DELTAE) then
    e = GEV250
elseif (circe1_params%roots .lt. 500d0 - DELTAE) then
    write (msgbuf, 2006) circe1_params%roots, 250d0, 500d0
    call circem ('MESSAGE', msgbuf)
    elo = GEV250
    ehi = GEV500
    eloal = 250d0
    ehival = 500d0
elseif (abs (circe1_params%roots-500d0) .le. DELTAE) then
    e = GEV500
elseif (circe1_params%roots .lt. 800d0 - DELTAE) then
    write (msgbuf, 2006) circe1_params%roots, 500d0, 800d0
    call circem ('MESSAGE', msgbuf)
    elo = GEV500
    ehi = GEV800
    eloal = 500d0
    ehival = 800d0
elseif (abs (circe1_params%roots-800d0) .le. DELTAE) then
    e = GEV800
elseif (circe1_params%roots .lt. 1000d0 - DELTAE) then
    write (msgbuf, 2006) circe1_params%roots, 800d0, 1000d0
    call circem ('MESSAGE', msgbuf)
    elo = GEV800
    ehi = TEV1
    eloal = 800d0
    ehival = 1000d0
elseif (abs (circe1_params%roots-1000d0) .le. DELTAE) then
    e = TEV1
elseif (circe1_params%roots .lt. 1200d0 - DELTAE) then
    write (msgbuf, 2006) circe1_params%roots, 1000d0, 1200d0
    call circem ('MESSAGE', msgbuf)
    elo = TEV1
    ehi = TEV12
    eloal = 1000d0
    ehival = 1200d0
elseif (abs (circe1_params%roots-1200d0) .le. DELTAE) then
    e = TEV12
elseif (circe1_params%roots .lt. 1500d0 - DELTAE) then
    write (msgbuf, 2006) circe1_params%roots, 1200d0, 1500d0
    call circem ('MESSAGE', msgbuf)
    elo = TEV12
    ehi = TEV15
    eloal = 1200d0

```

```

        ehival = 1500d0
    elseif (abs (circe1_params%roots-1500d0) .le. DELTAE) then
        e = TEV15
    else
        write (msgbuf, 2005) circe1_params%roots, 1500d0
        call circem ('MESSAGE', msgbuf)
        e = TEV15
    endif

```

Uses circem 87a.

65a \langle Linearly interpolate energies for NLC H 2002 65a $\rangle \equiv$

```

    e = GEV090 - 1
    elo = e
    ehi = e
    if (circe1_params%roots .lt. 500d0 - DELTAE) then
        write (msgbuf, 2004) circe1_params%roots, 500d0
        call circem ('MESSAGE', msgbuf)
        e = GEV500
    elseif (abs (circe1_params%roots-500d0) .le. DELTAE) then
        e = GEV500
    elseif (circe1_params%roots .lt. 1000d0 - DELTAE) then
        write (msgbuf, 2006) circe1_params%roots, 500d0, 1000d0
        call circem ('MESSAGE', msgbuf)
        elo = GEV500
        ehi = TEV1
        eloal = 500d0
        ehival = 1000d0
    elseif (abs (circe1_params%roots-1000d0) .le. DELTAE) then
        e = TEV1
    elseif (circe1_params%roots .lt. 1500d0 - DELTAE) then
        write (msgbuf, 2006) circe1_params%roots, 1000d0, 1500d0
        call circem ('MESSAGE', msgbuf)
        elo = TEV1
        ehi = TEV15
        eloal = 1000d0
        ehival = 1500d0
    elseif (abs (circe1_params%roots-1500d0) .le. DELTAE) then
        e = TEV15
    else
        write (msgbuf, 2005) circe1_params%roots, 1500d0
        call circem ('MESSAGE', msgbuf)
        e = TEV15
    endif

```

Uses circem 87a.

65b \langle Local variables for circses 33c $\rangle + \equiv$

```
integer, parameter :: A9NEGY = TEV15, A9NREV = 1
```

Note that ew *must not* interpolate a1(0) and a1(7) because they depend non-linearly on the other parameters!

65c \langle Update version 9 derived parameters in circe1 parameters 63b $\rangle + \equiv$

```

if (e .ge. GEV090) then
  circe1_params%lumi = xa9lum(e,circe1_params%acc,r)
  do i = 0, 7
    circe1_params%a1(i) = xa9(i,e,circe1_params%acc,r)
  end do
else if (elo .ge. GEV090 .and. ehi .ge. GEV090) then
  circe1_params%lumi = ((circe1_params%roots-eloal)*xa9lum(ehi,circe1_params%acc,r)
    + (ehival-circe1_params%roots)*xa9lum(elo,circe1_params%acc,r)) / (ehival - eloal)
  do i = 1, 6
    circe1_params%a1(i) = ((circe1_params%roots-eloal)*xa9(i,ehi,circe1_params%acc,r)
    + (ehival-circe1_params%roots)*xa9(i,elo,circe1_params%acc,r)) / (ehival - eloal)
  end do
  circe1_params%a1(0) = 1d0 - circe1_params%a1(1) * beta(circe1_params%a1(2)+1d0,
    circe1_params%a1(7) = circe1_params%a1(4) * beta(circe1_params%a1(5)+1d0,circe1_params%a1(7))
end if

```

Uses beta 105.

66a *<Local variables for circes 33c>+≡*
 real, dimension(GEV090:A9NEGY,NACC,0:A9NREV) :: xa9lum
 real, dimension(0:7,GEV090:A9NEGY,NACC,0:A9NREV) :: xa9

Uses NACC 13b.

Revision 1. The mother of all revisions.

66b *<Initializations for circes 35b>+≡*
 xa9lum(GEV090,TESLA,1) = -1.0
 xa9lum(GEV170,TESLA,1) = -1.0
 xa9lum(GEV350,TESLA,1) = -1.0
 xa9lum(GEV500,TESLA,1) = -1.0
 xa9lum(GEV800,TESLA,1) = -1.0
 xa9lum(TEV1, TESLA,1) = -1.0
 xa9lum(TEV12, TESLA,1) = -1.0
 xa9lum(TEV15, TESLA,1) = -1.0
 xa9lum(TEV16, TESLA,1) = -1.0

Uses TESLA 13a.

66c *<Initializations for circes 35b>+≡*
 xa9lum(GEV090,JLCNLC,1) = -1.0
 xa9lum(GEV170,JLCNLC,1) = -1.0
 xa9lum(GEV250,JLCNLC,1) = 109.886976
 xa9(0:7,GEV250,JLCNLC,1) = (/ &
 0.65598E+00, 0.34993E+00, 0.13766E+02,-0.64698E+00, &
 0.29984E+00,-0.69053E+00, 0.16444E+02, 0.36060E+00 /)
 xa9lum(GEV350,JLCNLC,1) = -1.0
 xa9lum(GEV500,JLCNLC,1) = 220.806144
 xa9(0:7,GEV500,JLCNLC,1) = (/ &
 0.57022E+00, 0.33782E+00, 0.52811E+01,-0.63540E+00, &
 0.32035E+00,-0.68776E+00, 0.69552E+01, 0.48751E+00 /)
 xa9lum(GEV800,JLCNLC,1) = 304.63488
 xa9(0:7,GEV800,JLCNLC,1) = (/ &
 0.54839E+00, 0.31823E+00, 0.33071E+01,-0.62671E+00, &
 0.31655E+00,-0.68468E+00, 0.45325E+01, 0.53449E+00 /)

```

xa9lum(TEV1, JLCNLC,1) = 319.95648
xa9(0:7,TEV1, JLCNLC,1) = (/ &
    0.56047E+00, 0.29479E+00, 0.28820E+01,-0.62856E+00, &
    0.29827E+00,-0.68423E+00, 0.39138E+01, 0.52297E+00 /)
xa9lum(TEV12,JLCNLC,1) = 349.90848
xa9(0:7,TEV12,JLCNLC,1) = (/ &
    0.56102E+00, 0.28503E+00, 0.24804E+01,-0.62563E+00, &
    0.29002E+00,-0.68376E+00, 0.33854E+01, 0.52736E+00 /)
xa9lum(TEV15,JLCNLC,1) = 363.15648
xa9(0:7,TEV15,JLCNLC,1) = (/ &
    0.57644E+00, 0.26570E+00, 0.22007E+01,-0.62566E+00, &
    0.27102E+00,-0.68283E+00, 0.29719E+01, 0.50764E+00 /)
xa9lum(TEV16,JLCNLC,1) = -1.0

```

Uses JLCNLC 13a.

67a *⟨Initializations for circes 35b⟩+≡*

```

xa9lum(GEV090,NLCH,1) = -1.0
xa9lum(GEV170,NLCH,1) = -1.0
xa9lum(GEV250,NLCH,1) = -1.0
xa9lum(GEV350,NLCH,1) = -1.0
xa9lum(GEV500,NLCH,1) = 371.4624
xa9(0:7,GEV500,NLCH,1)= (/ &
    0.33933E+00, 0.55165E+00, 0.29138E+01,-0.57341E+00, &
    0.54323E+00,-0.68590E+00, 0.51786E+01, 0.88956E+00 /)
xa9lum(GEV800,NLCH,1) = -1.0
xa9lum(TEV1,NLCH,1) = 516.41856
xa9(0:7,TEV1,NLCH,1)= (/ &
    0.35478E+00, 0.46474E+00, 0.17666E+01,-0.56949E+00, &
    0.49269E+00,-0.68384E+00, 0.31781E+01, 0.91121E+00 /)
xa9lum(TEV12,NLCH,1) = -1.0
xa9lum(TEV15,NLCH,1) = 575.06688
xa9(0:7,TEV15,NLCH,1)= (/ &
    0.38183E+00, 0.40310E+00, 0.13704E+01,-0.57742E+00, &
    0.44548E+00,-0.68341E+00, 0.24956E+01, 0.87448E+00 /)
xa9lum(TEV16,NLCH, 1) = -1.0

```

Revision 0.

67b *⟨Initializations for circes 35b⟩+≡*

```

xa9lum(GEV090,TESLA,0) = -1.0
xa9lum(GEV170,TESLA,0) = -1.0
xa9lum(GEV350,TESLA,0) = -1.0
xa9lum(GEV500,TESLA,0) = -1.0
xa9lum(GEV800,TESLA,0) = -1.0
xa9lum(TEV1, TESLA,0) = -1.0
xa9lum(TEV12, TESLA,0) = -1.0
xa9lum(TEV15, TESLA,0) = -1.0
xa9lum(TEV16, TESLA,0) = -1.0

```

Uses TESLA 13a.

67c *⟨Initializations for circes 35b⟩+≡*

```

xa9lum(GEV090,JLCNLC,0) = -1.0

```

```

xa9lum(GEV170,JLCNLC,0) = -1.0
xa9lum(GEV250,JLCNLC,0) = 109.886976
xa9(0:7,GEV250,JLCNLC,0) = (/ &
    0.65598E+00, 0.34993E+00, 0.13766E+02,-0.64698E+00, &
    0.29984E+00,-0.69053E+00, 0.16444E+02, 0.36060E+00 /)
xa9lum(GEV350,JLCNLC,0) = -1.0
xa9lum(GEV500,JLCNLC,0) = 220.806144
xa9(0:7,GEV500,JLCNLC,0) = (/ &
    0.57022E+00, 0.33782E+00, 0.52811E+01,-0.63540E+00, &
    0.32035E+00,-0.68776E+00, 0.69552E+01, 0.48751E+00 /)
xa9lum(GEV800,JLCNLC,0) = 304.63488
xa9(0:7,GEV800,JLCNLC,0) = (/ &
    0.54839E+00, 0.31823E+00, 0.33071E+01,-0.62671E+00, &
    0.31655E+00,-0.68468E+00, 0.45325E+01, 0.53449E+00 /)
xa9lum(TEV1, JLCNLC,0) = 319.95648
xa9(0:7,TEV1, JLCNLC,0) = (/ &
    0.56047E+00, 0.29479E+00, 0.28820E+01,-0.62856E+00, &
    0.29827E+00,-0.68423E+00, 0.39138E+01, 0.52297E+00 /)
xa9lum(TEV12,JLCNLC,0) = 349.90848
xa9(0:7,TEV12,JLCNLC,0) = (/ &
    0.56102E+00, 0.28503E+00, 0.24804E+01,-0.62563E+00, &
    0.29002E+00,-0.68376E+00, 0.33854E+01, 0.52736E+00 /)
xa9lum(TEV15,JLCNLC,0) = 363.15648
xa9(0:7,TEV15,JLCNLC,0) = (/ &
    0.57644E+00, 0.26570E+00, 0.22007E+01,-0.62566E+00, &
    0.27102E+00,-0.68283E+00, 0.29719E+01, 0.50764E+00 /)
xa9lum(TEV16,JLCNLC,0) = -1.0

```

Uses JLCNLC 13a.

68 *⟨Initializations for circes 35b⟩*+≡

```

xa9lum(GEV090,NLCH,0) = -1.0
xa9lum(GEV170,NLCH,0) = -1.0
xa9lum(GEV250,NLCH,0) = -1.0
xa9lum(GEV350,NLCH,0) = -1.0
xa9lum(GEV500,NLCH,0) = 371.4624
xa9(0:7,GEV500,NLCH,0) = (/ &
    0.33933E+00, 0.55165E+00, 0.29138E+01,-0.57341E+00, &
    0.54323E+00,-0.68590E+00, 0.51786E+01, 0.88956E+00 /)
xa9lum(GEV800,NLCH,0) = -1.0
xa9lum(TEV1,NLCH,0) = 516.41856
xa9(0:7,TEV1,NLCH,0) = (/ &
    0.35478E+00, 0.46474E+00, 0.17666E+01,-0.56949E+00, &
    0.49269E+00,-0.68384E+00, 0.31781E+01, 0.91121E+00 /)
xa9lum(TEV12,NLCH,0) = -1.0
xa9lum(TEV15,NLCH,0) = 575.06688
xa9(0:7,TEV15,NLCH,0) = (/ &
    0.38183E+00, 0.40310E+00, 0.13704E+01,-0.57742E+00, &
    0.44548E+00,-0.68341E+00, 0.24956E+01, 0.87448E+00 /)
xa9lum(TEV16,NLCH,0) = -1.0

```

6.2.9 Version 10

69a *<Update version 10 derived parameters in circe1 parameters 69a>*≡

```

    if (circe1_params%rev .eq. 0) then
        r = 0
    elseif (circe1_params%rev .ge. 20140305) then
        r = 1
    elseif (circe1_params%rev .lt. 20140305) then
        call circem ('ERROR', &
            'no revision of version 10 available before 2014/03/05')
        call circem ('MESSAGE', 'falling back to default')
        r = 1
    endif
    <Log revision mapping 38a>

```

Uses circem 87a.

69b *<Update version 10 derived parameters in circe1 parameters 69a>*+≡

```

    if (circe1_params%acc .ne. ILC) then
        call circem ('ERROR', 'version 10 applies to ILC only')
        call circem ('ERROR', 'falling back to ILC')
        circe1_params%acc = ILC
    end if
    if (circe1_params%acc .eq. ILC) then
        <Linearly interpolate energies for ILC 2013 69c>
    end if
    <Log energy mapping 39d>

```

Uses ILC 13a and circem 87a.

69c *<Linearly interpolate energies for ILC 2013 69c>*≡

```

    e = -EINVAL
    elo = -EINVAL
    ehi = -EINVAL
    if (circe1_params%roots .lt. 200d0 - DELTAE) then
        write (msgbuf, 2004) circe1_params%roots, 200d0
        call circem ('MESSAGE', msgbuf)
        e = GEV200
    elseif (abs (circe1_params%roots-200d0) .le. DELTAE) then
        e = GEV200
    elseif (circe1_params%roots .lt. 230d0 - DELTAE) then
        write (msgbuf, 2006) circe1_params%roots, 200d0, 230d0
        call circem ('MESSAGE', msgbuf)
        elo = GEV200
        ehi = GEV230
        eloval = 200d0
        ehival = 230d0
    elseif (abs (circe1_params%roots-230d0) .le. DELTAE) then
        e = GEV230
    elseif (circe1_params%roots .lt. 250d0 - DELTAE) then
        write (msgbuf, 2006) circe1_params%roots, 230d0, 250d0
        call circem ('MESSAGE', msgbuf)
        elo = GEV230
    end if

```

```

    ehi = GEV250
    eloal = 230d0
    ehival = 250d0
elseif (abs (circe1_params%roots-250d0) .le. DELTAE) then
    e = GEV250
elseif (circe1_params%roots .lt. 350d0 - DELTAE) then
    write (msgbuf, 2006) circe1_params%roots, 250d0, 350d0
    call circem ('MESSAGE', msgbuf)
    elo = GEV250
    ehi = GEV350
    eloal = 250d0
    ehival = 350d0
elseif (abs (circe1_params%roots-350d0) .le. DELTAE) then
    e = GEV350
elseif (circe1_params%roots .lt. 500d0 - DELTAE) then
    write (msgbuf, 2006) circe1_params%roots, 350d0, 500d0
    call circem ('MESSAGE', msgbuf)
    elo = GEV350
    ehi = GEV500
    eloal = 350d0
    ehival = 500d0
elseif (abs (circe1_params%roots-500d0) .le. DELTAE) then
    e = GEV500
else
    write (msgbuf, 2005) circe1_params%roots, 500d0
    call circem ('MESSAGE', msgbuf)
    e = GEV500
endif

```

Uses circem 87a.

70a \langle Local variables for circes 33c $\rangle + \equiv$

```
integer, parameter :: A1ONEGY = GEV230, A1ONREV = 1
```

Note that ew *must not* interpolate a1(0) and a1(7) because they depend non-linearly on the other parameters!

70b \langle Update version 10 derived parameters in circe1 parameters 69a $\rangle + \equiv$

```

if (e .ne. EINVAL) then
    circe1_params%lumi = xa10lum(e,circe1_params%acc,r)
    do i = 0, 7
        circe1_params%a1(i) = xa10(i,e,circe1_params%acc,r)
    end do
else if (elo .ne. EINVAL .and. ehi .ne. EINVAL) then
    circe1_params%lumi = ((circe1_params%roots-eloal)*xa10lum(ehi,circe1_params%acc,r)
        + (ehival-circe1_params%roots)*xa10lum(elo,circe1_params%acc,r)) / (ehival-eloal)
    do i = 1, 6
        circe1_params%a1(i) = ((circe1_params%roots-eloal)*xa10(i,ehi,circe1_params%acc,r)
            + (ehival-circe1_params%roots)*xa10(i,elo,circe1_params%acc,r)) / (ehival-eloal)
    end do
    circe1_params%a1(0) = 1d0 - circe1_params%a1(1) * beta(circe1_params%a1(2)+1d0,
        circe1_params%a1(7) = circe1_params%a1(4) * beta(circe1_params%a1(5)+1d0,circe1_params%a1(7))
end if

```

Uses **beta 105**.

```
71a <Local variables for circes 33c>+≡  
      real, dimension(GEV090:A1ONEGY,ILC:ILC,0:A1ONREV) :: xa10lum  
      real, dimension(0:7,GEV090:A1ONEGY,ILC:ILC,0:A1ONREV) :: xa10
```

Uses **ILC 13a**.

Revision 1. The mother of all revisions.

```
71b <Initializations for circes 35b>+≡  
      xa10lum = -1  
      xa10 = -1  
  
71c <Initializations for circes 35b>+≡  
      xa10lum(GEV200,ILC,1) = 56  
      xa10(:,GEV200,ILC,1) = (/ &  
          0.66253E+00, 0.51646E+00, 0.43632E+02, -0.64508E+00, &  
          0.35915E+00, -0.69716E+00, 0.51645E+02, 0.32097E+00 /)  
      xa10lum(GEV230,ILC,1) = 83  
      xa10(:,GEV230,ILC,1) = (/ &  
          0.62360E+00, 0.52780E+00, 0.31915E+02, -0.64171E+00, &  
          0.38375E+00, -0.69529E+00, 0.39717E+02, 0.36597E+00 /)  
      xa10lum(GEV250,ILC,1) = 97  
      xa10(:,GEV250,ILC,1) = (/ &  
          0.59996E+00, 0.52141E+00, 0.26647E+02, -0.64331E+00, &  
          0.39186E+00, -0.69687E+00, 0.33764E+02, 0.39669E+00 /)  
      xa10lum(GEV350,ILC,1) = 100  
      xa10(:,GEV350,ILC,1) = (/ &  
          0.58875E+00, 0.50027E+00, 0.18594E+02, -0.63380E+00, &  
          0.38659E+00, -0.69239E+00, 0.23964E+02, 0.42049E+00 /)  
      xa10lum(GEV500,ILC,1) = 180  
      xa10(:,GEV500,ILC,1) = (/ &  
          0.46755E+00, 0.51768E+00, 0.83463E+01, -0.62311E+00, &  
          0.45704E+00, -0.69165E+00, 0.12372E+02, 0.60192E+00 /)
```

Uses **ILC 13a**.

```
71d <Initializations for circes 35b>+≡
```

Revision 0 The latest is the default:

```
71e <Initializations for circes 35b>+≡  
      xa10lum(:, :, 0) = xa10lum(:, :, A1ONREV)  
      xa10(:, :, 0) = xa10(:, :, A1ONREV)
```

6.3 Special Functions

```
71f <Module subroutines 31b>+≡  
      function beta (a, b)  
          real(kind=double) :: a, b, beta  
          beta = exp (dlogam(a) + dlogam(b) - dlogam(a+b))  
      end function beta
```

Uses **beta 105**.

72 *<Module subroutines 31b>+≡*

!!! CERNLIB C304

```

function dlogam (x)
  real(kind=double) :: dlogam
  real(kind=double), dimension(7) :: p1, q1, p2, q2, p3, q3
  real(kind=double), dimension(5) :: c, xl
  real(kind=double) :: x, y, zero, one, two, half, ap, aq
  integer :: i
  data ZERO /0.0D0/, ONE /1.0D0/, TWO /2.0D0/, HALF /0.5D0/
  data XL /0.0D0,0.5D0,1.5D0,4.0D0,12.0D0/
  data p1 /+3.8428736567460D+0, +5.2706893753010D+1, &
    +5.5584045723515D+1, -2.1513513573726D+2, &
    -2.4587261722292D+2, -5.7500893603041D+1, &
    -2.3359098949513D+0/
  data q1 /+1.0000000000000D+0, +3.3733047907071D+1, &
    +1.9387784034377D+2, +3.0882954973424D+2, &
    +1.5006839064891D+2, +2.0106851344334D+1, &
    +4.5717420282503D-1/
  data p2 /+4.8740201396839D+0, +2.4884525168574D+2, &
    +2.1797366058896D+3, +3.7975124011525D+3, &
    -1.9778070769842D+3, -3.6929834005591D+3, &
    -5.6017773537804D+2/
  data q2 /+1.0000000000000D+0, +9.5099917418209D+1, &
    +1.5612045277929D+3, +7.2340087928948D+3, &
    +1.0459576594059D+4, +4.1699415153200D+3, &
    +2.7678583623804D+2/
  data p3 /-6.8806240094594D+3, -4.3069969819571D+5, &
    -4.7504594653440D+6, -2.9423445930322D+6, &
    +3.6321804931543D+7, -3.3567782814546D+6, &
    -2.4804369488286D+7/
  data q3 /+1.0000000000000D+0, -1.4216829839651D+3, &
    -1.5552890280854D+5, -3.4152517108011D+6, &
    -2.0969623255804D+7, -3.4544175093344D+7, &
    -9.1605582863713D+6/
  data c / 1.1224921356561D-1, 7.9591692961204D-2, &
    -1.7087794611020D-3, 9.1893853320467D-1, &
    1.3469905627879D+0/
  if (x .le. xl(1)) then
    print *, 'ERROR: DLOGAM non positive argument: ', X
    dlogam = zero
  end if
  if (x .le. xl(2)) then
    y = x + one
    ap = p1(1)
    aq = q1(1)
    do i = 2, 7
      ap = p1(i) + y * ap
      aq = q1(i) + y * aq
    end do

```

```

        y = - log(x) + x * ap / aq
    else if (x .le. xl(3)) then
        ap = p1(1)
        aq = q1(1)
        do i = 2, 7
            ap = p1(i) + x * ap
            aq = q1(i) + x * aq
        end do
        y = (x - one) * ap / aq
    else if (x .le. xl(4)) then
        ap = p2(1)
        aq = q2(1)
        do i = 2, 7
            ap = p2(i) + x * ap
            aq = q2(i) + x * aq
        end do
        y = (x-two) * ap / aq
    else if (x .le. xl(5)) then
        ap = p3(1)
        aq = q3(1)
        do i = 2, 7
            ap = p3(i) + x * ap
            aq = q3(i) + x * aq
        end do
        y = ap / aq
    else
        y = one / x**2
        y = (x-half) * log(x) - x + c(4) + &
            (c(1) + y * (c(2) + y * c(3))) / ((c(5) + y) * x)
    end if
    dlogam = y
end function dlogam

```

6.4 Non-Singular Distributions

73a *⟨Public subroutines 31a⟩*+≡

```
public :: kirke
```

Uses kirke **73b**.

73b *⟨Module subroutines 31b⟩*+≡

```

function kirke (x1, x2, p1, p2)
    real(kind=double) :: x1, x2
    real(kind=double) :: kirke
    integer :: p1, p2
    ⟨Initialization check 32h⟩
    kirke = -1.0
    if (abs(p1) .eq. C1_ELECTRON) then
        if (abs(p2) .eq. C1_ELECTRON) then
            kirke = kirkee (x1, x2)

```

```

        else if (p2 .eq. C1_PHOTON) then
            kirke = kirkeg (x1, x2)
        end if
    else if (p1 .eq. C1_PHOTON) then
        if (abs(p2) .eq. C1_ELECTRON) then
            kirke = kirkeg (x2, x1)
        else if (p2 .eq. C1_PHOTON) then
            kirke = kirkgg (x1, x2)
        end if
    endif
end function kirke

```

Defines:

kirke, used in chunk 73a.

Uses C1_ELECTRON 11b, C1_PHOTON 11b, kirkee 74b, kirkeg 76c, and kirkgg 77b.

74a \langle Public subroutines 31a $\rangle + \equiv$

```
public :: kirkee
```

Uses kirkee 74b.

74b \langle Module subroutines 31b $\rangle + \equiv$

```

function kirkee (x1, x2)
    real(kind=double) :: x1, x2
    real(kind=double) :: kirkee
    real(kind=double) :: d1, d2
     $\langle$ Initialization check 32h $\rangle$ 
    kirkee = -1.0
    if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
         $\langle$ Calculate version 1 of the non-singular  $e^+e^-$  distribution 75c $\rangle$ 
    else
         $\langle$ else handle invalid versions 37b $\rangle$ 
    end if
end function kirkee

```

Defines:

kirkee, used in chunks 17a and 73–75.

Uses d1 16a and d2 16c.

74c \langle 8-byte aligned part of circe1 parameters 32e $\rangle + \equiv$

```
real(kind=double) :: elect0, gamma0
```

$$\int_{1-\epsilon}^{1+} dx d_{e^\pm}^{\alpha_1 \rho}(x) = a_0^{\alpha \rho} + a_1^{\alpha \rho} \int_{1-\epsilon}^{1-} dx x^{a_2^{\alpha \rho}} (1-x)^{a_3^{\alpha \rho}} \quad (18)$$

Approximately

$$\int_{1-\epsilon}^{1+} dx d_{e^\pm}^{\alpha_1 \rho}(x) = a_0^{\alpha \rho} + a_1^{\alpha \rho} \int_{1-\epsilon}^{1-} dx (1-x)^{a_3^{\alpha \rho}} = a_0^{\alpha \rho} + a_1^{\alpha \rho} \int_{0+}^{\epsilon} d\xi \xi^{a_3^{\alpha \rho}} \quad (19)$$

and therefore

$$\int_{1-\epsilon}^{1+} dx d_{e^\pm}^{\alpha_1 \rho}(x) = a_0^{\alpha \rho} + a_1^{\alpha \rho} \frac{1 - \epsilon^{a_3^{\alpha \rho} + 1}}{a_3^{\alpha \rho} + 1} \quad (20)$$

This simple approximation is good enough

```

74d  <Update circe1 parameters 33b>+=
      circe1_params%elect0 = circe1_params%a1(0) + circe1_params%a1(1) * KIREPS**(circe1
      circe1_params%elect0 = circe1_params%elect0 / KIREPS
      circe1_params%gamma0 = circe1_params%a1(4) * KIREPS**(circe1_params%a1(5)+1) / (ci
      circe1_params%gamma0 = circe1_params%gamma0 / KIREPS

```

but we can also use incomplete Beta functions for the exact result:

```

75a  <Alternative: Update circe1 parameters 75a>≡
      circe1_params%elect0 = circe1_params%a1(0) + circe1_params%a1(1) * beta (circe1_pa
      * (1d0 - betinc (circe1_params%a1(2)+1, circe1_params%a1(3)+1, 1d0 -
      circe1_params%elect0 = circe1_params%elect0 / KIREPS
      circe1_params%gamma0 = circe1_params%a1(7) + circe1_params%a1(4) * beta (circe1_pa
      * betinc (circe1_params%a1(5)+1, circe1_params%a1(6)+1, KIREPS)
      circe1_params%gamma0 = circe1_params%gamma0 / KIREPS

```

Uses beta 105.

```

75b  <Alternative: Local variables for circes 75b>≡
      real(kind=double) :: betinc
      external betinc

```

```

75c  <Calculate version 1 of the non-singular  $e^+e^-$  distribution 75c>≡
      if (x1 .gt. 1d0) then
        d1 = 0d0
      elseif (x1 .ge. (1d0 - KIREPS)) then
        d1 = circe1_params%elect0
      elseif (x1 .ge. 0d0) then
        d1 = circe1_params%a1(1) * x1**circe1_params%a1(2) * (1d0 - x1)**circe1_params%
      else
        d1 = 0d0
      endif
      if (x2 .gt. 1d0) then
        d2 = 0d0
      elseif (x2 .ge. (1d0 - KIREPS)) then
        d2 = circe1_params%elect0
      elseif (x2 .ge. 0d0) then
        d2 = circe1_params%a1(1) * x2**circe1_params%a1(2) * (1d0 - x2)**circe1_params%
      else
        d2 = 0d0
      endif
      kirkee = d1 * d2

```

Uses d1 16a, d2 16c, and kirkee 74b.

```

75d  <Calculate version 1 of the non-singular  $e^\pm\gamma$  distribution 75d>≡
      if (x1 .gt. 1d0) then
        d1 = 0d0
      elseif (x1 .ge. (1d0 - KIREPS)) then
        d1 = circe1_params%elect0
      elseif (x1 .ge. 0d0) then
        d1 = circe1_params%a1(1) * x1**circe1_params%a1(2) * (1d0 - x1)**circe1_params%
      else
        d1 = 0d0
      endif

```

```

      if (x2 .gt. 1d0) then
        d2 = 0d0
      elseif (x2 .gt. KIREPS) then
        d2 = circe1_params%a1(4) * x2**circe1_params%a1(5) * (1d0 - x2)**circe1_params%
      elseif (x2 .ge. 0d0) then
        d2 = circe1_params%gamma0
      else
        d2 = 0d0
      endif
      kirkeg = d1 * d2

```

Uses d1 16a, d2 16c, and kirkeg 76c.

76a \langle Calculate version 1 of the non-singular $\gamma\gamma$ distribution 76a $\rangle \equiv$

```

      if (x1 .gt. 1d0) then
        d1 = 0d0
      elseif (x1 .gt. KIREPS) then
        d1 = circe1_params%a1(4) * x1**circe1_params%a1(5) * (1d0 - x1)**circe1_params%
      elseif (x1 .ge. 0d0) then
        d1 = circe1_params%gamma0
      else
        d1 = 0d0
      endif
      if (x2 .gt. 1d0) then
        d2 = 0d0
      elseif (x2 .gt. KIREPS) then
        d2 = circe1_params%a1(4) * x2**circe1_params%a1(5) * (1d0 - x2)**circe1_params%
      elseif (x2 .ge. 0d0) then
        d2 = circe1_params%gamma0
      else
        d2 = 0d0
      endif
      kirkgg = d1 * d2

```

Uses d1 16a, d2 16c, and kirkgg 77b.

76b \langle Public subroutines 31a $\rangle + \equiv$

```

      public :: kirkeg

```

Uses kirkeg 76c.

76c \langle Module subroutines 31b $\rangle + \equiv$

```

      function kirkeg (x1, x2)
        real(kind=double) :: x1, x2
        real(kind=double) :: kirkeg
        real(kind=double) :: d1, d2
         $\langle$ Initialization check 32h $\rangle$ 
        kirkeg = -1.0
        if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
           $\langle$ Calculate version 1 of the non-singular  $e^\pm\gamma$  distribution 75d $\rangle$ 
           $\langle$ else handle invalid versions 37b $\rangle$ 
        end if
      end function kirkeg

```

Defines:

kirkeg, used in chunks 73b, 75d, and 76b.
 Uses d1 16a and d2 16c.

77a *<Public subroutines 31a>+≡*
 public :: kirkeg

Uses kirkeg 77b.

77b *<Module subroutines 31b>+≡*
 function kirkeg (x1, x2)
 real(kind=double) :: x1, x2
 real(kind=double) :: kirkeg
 real(kind=double) :: d1, d2
 <Initialization check 32h>
 kirkeg = -1.0
 if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
 <Calculate version 1 of the non-singular $\gamma\gamma$ distribution 76a>
 else handle invalid versions 37b
 end function kirkeg

Defines:

kirkeg, used in chunks 73b, 76a, and 77a.
 Uses d1 16a and d2 16c.

77c *<Alternative: Subroutines 77c>≡*
 function betinc (a, b, x)
 real(kind=double) :: x, a, b
 real(kind=double) :: betinc
 real(kind=double) :: bt
 if (x .lt. 0d0 .or. x .gt. 1d0) then
 betinc = 0d0
 else
 if (x .eq. 0d0 .or. x .eq. 1d0) then
 bt = 0d0
 else
 bt = exp(dlogam(a+b)-dlogam(a)-dlogam(b) &
 + a*log(x) + b*log(1d0-x))
 end if
 if (x .lt. (a+1d0)/(a+b+2d0)) then
 betinc = bt*betacf (a, b, x) / a
 else
 betinc = 1d0 - bt*betacf (b, a, 1d0-x) / b
 end if
 end if
 end function betinc

77d *<Alternative: Subroutines 77c>+≡*
 function betacf (a, b, x)
 real(kind=double) :: x, a, b
 real(kind=double) :: betacf
 integer, parameter :: itmax = 100
 real(kind=double), parameter = eps = 3d-7

```

real(kind=double) :: am, bm, curr, prev, qab, qap, qam, bz, &
                    ap, bp, app, bpp, em, tem, d

integer :: m
am = 1d0
bm = 1d0
curr = 1d0
qab = a + b
qap = a + 1d0
qam = a - 1d0
bz = 1d0 - qab * x / qap
do m = 1, ITMAX
    em = m
    tem = 2*em
    d = em * (b - m) * x / ((qam + tem) * (a + tem))
    ap = curr + d*am
    bp = bz + d*bm
    d = - (a + em) * (qab + em) * x / ((a + tem) * (qap + tem))
    app = ap + d * curr
    bpp = bp + d * bz
    prev = curr
    am = ap / bpp
    bm = bp / bpp
    curr = app / bpp
    bz = 1d0
    if (abs (curr - prev) .lt. EPS * abs (curr)) then
        betacf = curr
        return
    end if
end do
print *, 'betacf: failed to converge'
betacf = 0d0
end

```

6.5 Generators

6.5.1 Random-Number Generator

The generator routines do not fix or provide a random-number generator. The caller has to provide an implementation which is transferred to the subroutines in one of two possible forms:

1. as a subroutine which generates a single random number, working on an implicit external state
2. as an object with a method the generates a single random number, working on an internal state

These snippets should be used by the procedures that use a RNG:

78 *(RNG dummy arguments 78)* \equiv
 rng, rng_obj

79a *⟨RNG dummy declarations 79a⟩*≡

```

    procedure(rng_proc), optional :: rng
    class(rng_type), intent(inout), optional :: rng_obj

```

 Uses rng_proc **79d** and rng_type **79f**.

79b *⟨RNG: generate u 79b⟩*≡

```

    call rng_call (u, ⟨RNG dummy arguments 78⟩)

```

 Uses rng_call **79c**.

79c *⟨Module subroutines 31b⟩*+≡

```

    subroutine rng_call (u, ⟨RNG dummy arguments 78⟩)
    real(kind=double), intent(out) :: u
    ⟨RNG dummy declarations 79a⟩
    if (present (rng)) then
        call rng (u)
    else if (present (rng_obj)) then
        call rng_obj%generate (u)
    else
        call circem ('PANIC', &
            'generator requires either rng or rng_obj argument')
    end if
end subroutine rng_call

```

Defines:

 rng_call, used in chunk **79b**.

Uses circem **87a**.

This defines the procedure version of the RNG, corresponding to the traditional F77 external interface. The abstract interface enables the compiler to check conformance.

79d *⟨Abstract interfaces 79d⟩*≡

```

    abstract interface
    subroutine rng_proc (u)
    import :: double
    real(kind=double), intent(out) :: u
    end subroutine rng_proc
end interface

```

Defines:

 rng_proc, used in chunk **79**.

Here we define the object version of the RNG. It has to implement a generate method which parallels the rng_proc procedure above.

79e *⟨Public types 79e⟩*≡

```

    public :: rng_type

```

 Uses rng_type **79f**.

79f *⟨Abstract types 79f⟩*≡

```

    type, abstract :: rng_type
    contains
    procedure(rng_generate), deferred :: generate
end type rng_type

```

Defines:

`rng_type`, used in chunks 79 and 80a.

Uses `rng_generate` 80a.

```
80a <Abstract interfaces 79d>+≡
    abstract interface
        subroutine rng_generate (rng_obj, u)
            import :: rng_type, double
            class(rng_type), intent(inout) :: rng_obj
            real(kind=double), intent(out) :: u
        end subroutine rng_generate
    end interface
```

Defines:

`rng_generate`, used in chunk 79f.

Uses `rng_type` 79f.

6.5.2 Version 1

Beta distributions have the practical advantage that they have been popular among mathematicians.[?]

```
80b <Public subroutines 31a>+≡
    public :: girce
```

Uses `girce` 80c.

```
80c <Module subroutines 31b>+≡
    subroutine girce (x1, x2, p1, p2, <RNG dummy arguments 78>)
        real(kind=double), intent(out) :: x1, x2
        integer :: p1, p2
        <RNG dummy declarations 79a>
        real(kind=double) :: u, w
        <Initialization check 32h>
        <x1m, x2m kludge, part 1 81b>
        <Select particles p1 and p2 81a>
        if (abs(p1) .eq. C1_ELECTRON) then
            if (abs(p2) .eq. C1_ELECTRON) then
                call gircee (x1, x2, <RNG dummy arguments 78>)
            else if (p2 .eq. C1_PHOTON) then
                call girceg (x1, x2, <RNG dummy arguments 78>)
            end if
        else if (p1 .eq. C1_PHOTON) then
            if (abs(p2) .eq. C1_ELECTRON) then
                call girceg (x2, x1, <RNG dummy arguments 78>)
            else if (p2 .eq. C1_PHOTON) then
                call gircgg (x1, x2, <RNG dummy arguments 78>)
            end if
        end if
        <x1m, x2m kludge, part 2 81c>
    end subroutine girce
```

Defines:

gircee, used in chunks 80c, 20a, and 80.
 Uses C1_ELECTRON 11b, C1_PHOTON 11b, gircee 81e, girceg 82c, and girceg 83c.

```
81a <Select particles p1 and p2 81a>≡
      w = 1d0 / (1d0 + circgg (-1d0, -1d0))
      <RNG: generate u 79b>
      if (u*u .le. w) then
        p1 = C1_POSITRON
      else
        p1 = C1_PHOTON
      end if
      <RNG: generate u 79b>
      if (u*u .le. w) then
        p2 = C1_ELECTRON
      else
        p2 = C1_PHOTON
      end if
```

Uses C1_ELECTRON 11b, C1_PHOTON 11b, C1_POSITRON 11b, and circgg 43c.

The flavor selection is incorrect, because the relative weights depend on the minimum energy fractions. We resort to a moderately inefficient kludge, because we don't have the distribution functions available yet. We'll have to implement incomplete Beta functions and other horrible things for this. Fortunately, the efficiency can not drop below the relative contribution of e^+e^- .

```
81b <x1m, x2m kludge, part 1 81b>≡
      do
```

Crude rejection:

```
81c <x1m, x2m kludge, part 2 81c>≡
      if ((x1 .ge. circe1_params%x1m) .and. (x2 .ge. circe1_params%x2m)) exit
      end do
```

```
81d <Public subroutines 31a>+≡
      public :: gircee
```

Uses gircee 81e.

```
81e <Module subroutines 31b>+≡
      subroutine gircee (x1, x2, <RNG dummy arguments 78>)
        real(kind=double), intent(out) :: x1, x2
        <RNG dummy declarations 79a>
        real(kind=double) :: u
        <Initialization check 32h>
        x1 = 1
        x2 = 1
        if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
          <Generate version 1 of the  $e^+e^-$  distribution 82a>
          <else handle invalid versions 37b>
        end subroutine gircee
```

Defines:

gircee, used in chunks 20, 81e, 21d, 80c, and 81d.

For version 1 of the parametrizations we rely on `girceb`, a fast generator of β -distributions:

$$\beta_{x_{\min}, x_{\max}}^{a,b}(x) = x^{a-1}(1-x)^{b-1} \cdot \frac{\Theta(x_{\max}-x)\Theta(x-x_{\min})}{I(x_{\min}, a, b) - I(x_{\max}, a, b)} \quad (21)$$

$$I(x, a, b) = \int_x^1 d\xi \xi^{a-1}(1-\xi)^{b-1} \quad (22)$$

```

82a  <Generate version 1 of the  $e^+e^-$  distribution 82a>≡
      <RNG: generate u 79b>
      if (u .le. circe1_params%a1(0)) then
        x1 = 1d0
      else
        x1 = 1d0 - girceb (0d0, 1d0-circe1_params%x1m, &
                           circe1_params%a1(3)+1d0, circe1_params%a1(2)+1d0, &
                           <RNG dummy arguments 78>)
      endif
      <RNG: generate u 79b>
      if (u .le. circe1_params%a1(0)) then
        x2 = 1d0
      else
        x2 = 1d0 - girceb (0d0, 1d0-circe1_params%x2m, &
                           circe1_params%a1(3)+1d0, circe1_params%a1(2)+1d0, &
                           <RNG dummy arguments 78>)
      endif
      Uses girceb 84b.

82b  <Public subroutines 31a>+≡
      public :: girceg
      Uses girceg 82c.

82c  <Module subroutines 31b>+≡
      subroutine girceg (x1, x2, <RNG dummy arguments 78>)
        real(kind=double), intent(out) :: x1, x2
        <RNG dummy declarations 79a>
        real(kind=double) :: u
        <Initialization check 32h>
        x1 = 1
        x2 = 1
        if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
          <Generate version 1 of the  $e^\pm\gamma$  distribution 82d>
          <else handle invalid versions 37b>
        end subroutine girceg

```

Defines:

`girceg`, used in chunks 20c, 80c, and 82b.

```

82d  <Generate version 1 of the  $e^\pm\gamma$  distribution 82d>≡
      <RNG: generate u 79b>
      if (u .le. circe1_params%a1(0)) then
        x1 = 1d0

```

```

else
  x1 = 1d0 - girceb (0d0, 1d0-circe1_params%x1m, &
                    circe1_params%a1(3)+1d0, circe1_params%a1(2)+1d0, &
                    <RNG dummy arguments 78>)
endif
x2 = girceb (circe1_params%x2m, 1d0, &
            circe1_params%a1(5)+1d0, circe1_params%a1(6)+1d0, &
            <RNG dummy arguments 78>)

```

Uses girceb 84b.

83a <Public subroutines 31a>+≡

```

public :: gircgg

```

Uses gircgg 83c.

83b <Module subroutines 31b>+≡

```

subroutine gircgg (x1, x2, <RNG dummy arguments 78>)
  real(kind=double), intent(out) :: x1, x2
  <RNG dummy declarations 79a>
  <Initialization check 32h>
  x1 = 1
  x2 = 1
  if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
    <Generate version 1 of the γγ distribution 83c>
  <else handle invalid versions 37b>
  end subroutine gircgg

```

Uses gircgg 83c.

83c <Generate version 1 of the γγ distribution 83c>≡

```

x1 = girceb (circe1_params%x1m, 1d0, &
            circe1_params%a1(5)+1d0, circe1_params%a1(6)+1d0, &
            <RNG dummy arguments 78>)
x2 = girceb (circe1_params%x2m, 1d0, &
            circe1_params%a1(5)+1d0, circe1_params%a1(6)+1d0, &
            <RNG dummy arguments 78>)

```

Defines:

gircgg, used in chunks 20c, 80c, and 83.

Uses girceb 84b.

6.5.3 Version 2

Retired.

6.5.4 Version 3 and 4

Identical to version 1.

6.6 Utilities

For version 1 of the parametrizations we need a fast generator of β -distributions:

$$\beta_{x_{\min}, x_{\max}}^{a,b}(x) = x^{a-1}(1-x)^{b-1} \cdot \frac{\Theta(x_{\max}-x)\Theta(x-x_{\min})}{I(x_{\min}, a, b) - I(x_{\max}, a, b)} \quad (23)$$

with the *incomplete Beta-function* I :

$$I(x, a, b) = \int_x^1 d\xi \xi^{a-1} (1 - \xi)^{b-1} \quad (24)$$

$$B(a, b) = I(0, a, b) \quad (25)$$

This problem has been studied extensively [?] and we can use an algorithm [18] that is very fast for $0 < a \leq 1 \leq b$, which turns out to be the case in our application.

84a \langle Public subroutines 31a $\rangle + \equiv$

public :: girceb

Uses girceb 84b.

84b \langle Module subroutines 31b $\rangle + \equiv$

function girceb (xmin, xmax, a, b, \langle RNG dummy arguments 78 \rangle)

real(kind=double) :: xmin, xmax, a, b

real(kind=double) :: girceb

\langle RNG dummy declarations 79a \rangle

real(kind=double) :: t, p, u, umin, umax, x, w

\langle Check a and b 84c \rangle

\langle Set up girceb parameters 85a \rangle

do

\langle Generate a trial x and calculate its weight w 85b \rangle

\langle RNG: generate u 79b \rangle

if (w .gt. u) exit

end do

girceb = x

end function girceb

Defines:

girceb, used in chunks 84b, 82–84, and 86b.

In fact, this algorithm works for $0 < a \leq 1 \leq b$ only:

84c \langle Check a and b 84c $\rangle \equiv$

if ((a .ge. 1d0) .or. (b .le. 1d0)) then

girceb = -1d0

call circem ('ERROR', 'beta-distribution expects a<1<b')

return

end if

Uses circem 87a and girceb 84b.

The trick is to split the interval $[0, 1]$ into two parts $[0, t]$ and $[t, 1]$. In these intervals we obviously have

$$x^{a-1}(1-x)^{b-1} \leq \begin{cases} x^{a-1} & \text{for } x \leq t \\ t^{a-1}(1-x)^{b-1} & \text{for } x \geq t \end{cases} \quad (26)$$

because we have assumed that $0 < a < 1 < b$. The integrals of the two dominating distributions are t^a/a and $t^{a-1}(1-t)^b/b$ respectively and therefore the probability for picking a random number from the first interval is

$$P(x \leq t) = \frac{bt}{bt + a(1-t)^b} \quad (27)$$

We postpone the discussion of the choice of t until later:

```
85a <Set up girceb parameters 85a>≡
    <Set up best value for t 86c>
    p = b*t / (b*t + a * (1d0 - t)**b)
```

The dominating distributions can be generated by simple mappings

$$\phi : [0, 1] \rightarrow [0, 1] \quad (28)$$

$$u \mapsto \begin{cases} t \left(\frac{u}{p} \right)^{\frac{1}{a}} & < t \text{ for } u < p \\ t & = t \text{ for } u = p \\ 1 - (1 - t) \left(\frac{1-u}{1-p} \right)^{\frac{1}{b}} & > t \text{ for } u > p \end{cases} \quad (29)$$

The beauty of the algorithm is that we can use a single uniform deviate u for both intervals:

```
85b <Generate a trial x and calculate its weight w 85b>≡
    <RNG: generate u 79b>
    u = umin + (umax - umin) * u
    if (u .le. p) then
        x = t * (u/p)**(1d0/a)
        w = (1d0 - x)**(b-1d0)
    else
        x = 1d0 - (1d0 - t) * ((1d0 - u)/(1d0 - p))**(1d0/b)
        w = (x/t)**(a-1d0)
    end if
```

The weights that are derived by dividing the distribution by the dominating distributions are already normalized correctly:

$$w : [0, 1] \rightarrow [0, 1] \quad (30)$$

$$x \mapsto \begin{cases} (1 - x)^{b-1} & \in [(1 - t)^{b-1}, 1] \text{ for } x \leq t \\ \left(\frac{x}{t} \right)^{a-1} & \in [t^{1-a}, 1] \text{ for } x \geq t \end{cases} \quad (31)$$

To derive $u_{\min, \max}$ from $x_{\min, \max}$ we can use ϕ^{-1} :

$$\phi^{-1} : [0, 1] \rightarrow [0, 1] \quad (32)$$

$$x \mapsto \begin{cases} p \left(\frac{x}{t} \right)^a & < p \text{ for } x < t \\ p & = p \text{ for } x = t \\ 1 - (1 - p) \left(\frac{1-x}{1-t} \right)^b & > p \text{ for } x > t \end{cases} \quad (33)$$

We start with u_{\min} . For efficiency, we handle the most common cases (small x_{\min}) first:

```
85c <Set up girceb parameters 85a>+≡
    if (xmin .le. 0d0) then
        umin = 0d0
    elseif (xmin .lt. t) then
        umin = p * (xmin/t)**a
    elseif (xmin .eq. t) then
        umin = p
```

```

elseif (xmin .lt. 1d0) then
  umin = 1d0 - (1d0 - p) * ((1d0 - xmin)/(1d0 - t))**b
else
  umin = 1d0
endif

```

Same procedure for u_{\max} ; again, handle the most common cases (large x_{\max}) first:

```

86a <Set up girceb parameters 85a>+≡
  if (xmax .ge. 1d0) then
    umax = 1d0
  elseif (xmax .gt. t) then
    umax = 1d0 - (1d0 - p) * ((1d0 - xmax)/(1d0 - t))**b
  elseif (xmax .eq. t) then
    umax = p
  elseif (xmax .gt. 0d0) then
    umax = p * (xmax/t)**a
  else
    umax = 0d0
  endif

```

Check for absurd cases.

```

86b <Set up girceb parameters 85a>+≡
  if (umax .lt. umin) then
    girceb = -1d0
    return
  endif

```

Uses `girceb 84b`.

It remains to choose the best value for t . The rejection efficiency ϵ of the algorithm is given by the ratio of the dominating distribution and the distribution

$$\frac{1}{\epsilon(t)} = \frac{B(a, b)}{ab} (bt^a + at^{a-1}(1-t)^b). \quad (34)$$

It is maximized for

$$bt - bt(1-t)^{b-1} + (a-1)(1-t)^b = 0 \quad (35)$$

This equation has a solution which can be determined numerically. While this determination is far too expensive compared to a moderate loss in efficiency, we could perform it once after fitting the coefficients a, b . Nevertheless, it has been shown,[\[18\]](#) that

$$t = \frac{1-a}{b+1-a} \quad (36)$$

results in non-vanishing efficiency for all values $1 < a \leq 1 \leq b$. Empirically we have found efficiencies of at least 80% for this choice, which is enough for our needs.

```

86c <Set up best value for t 86c>≡
  t = (1d0 - a) / (b + 1d0 - a)

```

```

86d <Public subroutines 31a>+≡
  public :: circem

```

Uses `circem 87a`.

87a *<Module subroutines 31b>+≡*

```

subroutine circem (errlvl, errmsg)
  character(len=*) :: errlvl, errmsg
  integer, save :: errcnt = 0
  if (errlvl .eq. 'MESSAGE') then
    print *, 'circe1:message: ', errmsg
  else if (errlvl .eq. 'WARNING') then
    if (errcnt .lt. 100) then
      errcnt = errcnt + 1
      print *, 'circe1:warning: ', errmsg
    else if (errcnt .eq. 100) then
      errcnt = errcnt + 1
      print *, 'circe1:message: more than 100 messages'
      print *, 'circe1:message: turning warnings off'
    end if
  else if (errlvl .eq. 'ERROR') then
    if (errcnt .lt. 200) then
      errcnt = errcnt + 1
      print *, 'circe1:error: ', errmsg
    else if (errcnt .eq. 200) then
      errcnt = errcnt + 1
      print *, 'circe1:message: more than 200 messages'
      print *, 'circe1:message: turning error messages off'
    endif
  else if (errlvl .eq. 'PANIC') then
    if (errcnt .lt. 300) then
      errcnt = errcnt + 1
      print *, 'circe1:panic: ', errmsg
    else if (errcnt .eq. 300) then
      errcnt = errcnt + 1
      print *, 'circe1:message: more than 300 messages'
      print *, 'circe1:message: turning panic messages off'
    end if
  else
    print *, 'circe1:panic: invalid error code ', errlvl
  end if
end subroutine circem

```

Defines:

`circem`, used in chunks 33, 34, 36–39, 50, 51, 53–55, 57, 61, 63, 65a, 69, 79c, 84c, and 86d.

6.7 Examples

6.7.1 Distributions

87b *<circe1_plot.f90 87b>≡*

```

program circe1_plot
  use kinds
  use circe1

```

```

implicit none

real(kind=double) :: xmin, xmax, y, roots
integer :: xory, nstep, p1, p2, acc, ver, rev, i
real(kind=double) :: x, logx, d
read *, xory, xmin, xmax, nstep, y, p1, p2, roots, acc, ver, rev
call circes (0d0, 0d0, roots, acc, ver, rev, 0)
do i = 0, nstep
  logx = log (xmin) + i * log (xmax/xmin) / nstep
  x = exp (logx)
  d = 0d0
  if (xory .eq. 1) then
    if (p1 .eq. C1_PHOTON) then
      d = circe (x, y, p1, p2)
    else
      d = circe (1d0 - x, y, p1, p2)
    end if
  else if (xory .eq. 2) then
    if (p1 .eq. C1_PHOTON) then
      d = circe (y, x, p1, p2)
    else
      d = circe (y, 1d0 - x, p1, p2)
    end if
  end if
  if (d .gt. 1d-4) print *, x, d
end do
end program circe1_plot

```

Uses C1_PHOTON 11b, circe 31b, and circes 32b.

6.7.2 Library functions

If Fortran77 only had first class functions, then the following cruft would not be necessary. OK, here's the outline of the adaptive Gauss integration routine from CERNLIB:

```

88 <Part one of Gaussian integration 88>≡
  real(kind=double) :: f, a, b, eps
  external f
  real(kind=double), parameter :: Z1 = 1, HF = Z1/2, CST = 5*Z1/1000
  integer :: i
  real(kind=double) :: h, const, aa, bb, c1, c2, s8, s16, u
  <Gaussian weights 91a>
  h = 0
  if (b .eq. a) go to 99
  const = CST / dabs(b-a)
  bb = a
1 continue
  aa = bb
  bb = b

```

```

2 continue
  c1 = HF*(bb+aa)
  c2 = HF*(bb-aa)
  s8 = 0
  do i = 1, 4
    u = c2*x(i)

```

Here are now the first two function calls that we have to fill in later in various ways:

```

89a <Function call stub 89a>≡
      s8 = s8 + w(i) * (f (c1+u) + f (c1-u))

```

Continuing

```

89b <Part two of Gaussian integration 89b>≡
      end do
      s16 = 0
      do i = 5, 12
        u = c2*x(i)

```

And here are the other two function calls:

```

89c <Function call stub 89a>+≡
      s16 = s16 + w(i) * (f (c1+u) + f (c1-u))

```

Terminating:

```

89d <Part three of Gaussian integration 89d>≡
      end do
      s16 = c2*s16
      if (dabs(s16-c2*s8) .le. eps*(1+dabs(s16))) then
        h = h + s16
        if (bb .ne. b) go to 1
      else
        bb = c1
        if (1 + const*dabs(c2) .ne. 1) go to 2
        h = 0
        print *, 'gauss: too high accuracy required'
        go to 99
      end if
      99 continue

```

This one is still reasonably straightforward

$$\text{gauss1} : (f, a, b) \mapsto \int_a^b dx f(x) \quad (37)$$

```

89e <circe1_sample.f90: public 15a>+≡
  public :: gauss1

```

Uses gauss1 89f.

```

89f <circe1_sample.f90: subroutines 15b>+≡
  function gauss1 (f, a, b, eps)
    real(kind=double) :: gauss1
    <Part one of Gaussian integration 88>
      s8 = s8 + w(i) * (f (c1+u) + f (c1-u))
    <Part two of Gaussian integration 89b>

```

```

s16 = s16 + w(i) * (f (c1+u) + f (c1-u))
  ⟨Part three of Gaussian integration 89d⟩
gauss1 = h
end function gauss1

```

Defines:

`gauss1`, used in chunks 89f, 15c, and 89.

But this almost identical repeat

$$\text{gaussx} : (f, a, b) \mapsto \left(y \mapsto \int_a^b dx f(y, x) \right) \quad (38)$$

would not be necessary in a modern programming language with currying:

```

90a ⟨circe1_sample.f90: public 15a⟩+≡
    public :: gaussx

```

Uses `gaussx` 90b.

```

90b ⟨circe1_sample.f90: subroutines 15b⟩+≡
    function gaussx (f, y, a, b, eps)
      real(kind=double) :: y
      real(kind=double) :: gaussx
    ⟨Part one of Gaussian integration 88⟩
      s8 = s8 + w(i) * (f (y, c1+u) + f (y, c1-u))
    ⟨Part two of Gaussian integration 89b⟩
      s16 = s16 + w(i) * (f (y, c1+u) + f (y, c1-u))
    ⟨Part three of Gaussian integration 89d⟩
      gaussx = h
    end function gaussx

```

Defines:

`gaussx`, used in chunk 90.

Fortunately, this is the last one we need

$$\begin{aligned} \text{gauss2} : (f, a, b, a_1, b_1) \mapsto \int_a^b dx \int_{a_1}^{b_1} dy f(x, y) \\ = \text{gauss1}(\text{gaussx}(f, a, b), a_1, b_1) \end{aligned} \quad (39)$$

```

90c ⟨circe1_sample.f90: public 15a⟩+≡
    public :: gauss2

```

Uses `gauss2` 90d.

```

90d ⟨circe1_sample.f90: subroutines 15b⟩+≡
    function gauss2 (f, a, b, a1, b1, eps)
      real(kind=double) :: a1, b1
      real(kind=double) :: gauss2
    ⟨Part one of Gaussian integration 88⟩
      s8 = s8 + w(i) * (gaussx (f, c1+u, a1, b1, eps) &
        + gaussx (f, c1-u, a1, b1, eps))
    ⟨Part two of Gaussian integration 89b⟩
      s16 = s16 + w(i) * (gaussx (f, c1+u, a1, b1, eps) &

```

```

+ gaussx (f, c1-u, a1, b1, eps))
<Part three of Gaussian integration 89d>
  gauss2 = h
end function gauss2

```

Defines:

gauss2, used in chunks 90d, 15c, 16e, and 90.

Uses gaussx 90b.

```

91a <Gaussian weights 91a>≡
  real(kind=double), dimension(12), parameter :: &
    x = (/ 9.6028985649753623d-1, &
          7.9666647741362674d-1, &
          5.2553240991632899d-1, &
          1.8343464249564980d-1, &
          9.8940093499164993d-1, &
          9.4457502307323258d-1, &
          8.6563120238783174d-1, &
          7.5540440835500303d-1, &
          6.1787624440264375d-1, &
          4.5801677765722739d-1, &
          2.8160355077925891d-1, &
          9.5012509837637440d-2 /), &
    w = (/ 1.0122853629037626d-1, &
          2.2238103445337447d-1, &
          3.1370664587788729d-1, &
          3.6268378337836198d-1, &
          2.7152459411754095d-2, &
          6.2253523938647893d-2, &
          9.5158511682492785d-2, &
          1.2462897125553387d-1, &
          1.4959598881657673d-1, &
          1.6915651939500254d-1, &
          1.8260341504492359d-1, &
          1.8945061045506850d-1 /)

```

6.7.3 Generators

6.8 Dumping Parameters

```

91b <params.f90 91b>≡
  program params
    use kinds
    use circe1

    implicit none
    integer :: acc, ver, i
    real(kind=double), dimension(7), parameter :: roots = &
      (/ 90D0, 170D0, 350D0, 500D0, 800D0, 1000D0, 1500D0 /)
    do ver = 7, 8

```

```

        print *, "VERSION ", ver
        do acc = TESLA, XBNDEE
            do 12 i = 1, 7
                print *, "=====
                call circes (0d0, 0d0, roots(i), acc, ver, 20020307, 0)
                call dump ()
            end do
        end do
    end do
end program params

```

Uses TESLA 13a, XBNDEE 13a, and circes 32b.

```

92 <params.f90 91b>+≡
    subroutine dump
        <Accelerator codes 13a>
        character(len=9) :: name
        select case (acc)
            case (SBAND)
                name = 'SBAND'
            case (TESLA)
                name = 'TESLA'
            case (JLCNLC)
                name = 'JLCNLC'
            case (SBNDEE)
                name = 'SBAND/EE'
            case (TESLEE)
                name = 'TESLA/EE'
            case (XBNDEE)
                name = 'JLCNLC/EE'
            case (ILC)
                name = 'ILC'
            case default
                print *, "Accelerator mode not recognized"
        end select
        write (*, 1000) name, circe1_params%roots
        write (*, 1001) 'e+/e-', circe1_params%lumi
        write (*, 1002) 'e+/e-', circe1_params%a1(0)
        write (*, 1003) 'e+/e-', 1 - circe1_params%a1(0)
        write (*, 1004) 'e+/e-', circe1_params%a1(1), circe1_params%a1(2), circe1_params%a1(3)
        write (*, 1003) 'gamma', circe1_params%a1(7)
        write (*, 1004) 'gamma', circe1_params%a1(4), circe1_params%a1(5), circe1_params%a1(6)
        1000 format (A9, ' @ ', F5.0, ' GeV')
        1001 format (4X, A7, ' lumi          = ', F7.2, ' * 1032 cm-2 sec-1')
        1002 format (4X, A7, ' delta strength = ', F9.5)
        1003 format (4X, A7, ' integral(cont.) = ', F9.5)
        1004 format (4X, A7, ' distribution   = ', F9.5, ' * x{', F9.5, ' } * (1-x){', F9.5, ' }
        end subroutine dump

```

Uses ILC 13a, JLCNLC 13a, SBAND 13a, SBNDEE 13a, TESLA 13a, TESLEE 13a, and XBNDEE 13a.

7 Fitting

7.1 Version 1: Factorized Beta Distributions

```
93a <Copyleft notice 30a>+≡
!
! Copyright (C) 1999-2014 by
!   Wolfgang Kilian <kilian@physik.uni-siegen.de>
!   Thorsten Ohl <ohl@physik.uni-wuerzburg.de>
!   Juergen Reuter <juergen.reuter@desy.de>
!   Christian Speckner <cnspeckn@googlemail.com>
!
! WHIZARD is free software; you can redistribute it and/or modify it
! under the terms of the GNU General Public License as published by
! the Free Software Foundation; either version 2, or (at your option)
! any later version.
!
! WHIZARD is distributed in the hope that it will be useful, but
! WITHOUT ANY WARRANTY; without even the implied warranty of
! MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
! GNU General Public License for more details.
!
! You should have received a copy of the GNU General Public License
! along with this program; if not, write to the Free Software
! Foundation, Inc., 675 Mass Ave, Cambridge, MA 02139, USA.
!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
! This file has been stripped of most comments. For documentation, refer
! to the source 'minuit.nw'

93b <circe1_fit.f90 93b>≡
! circe1_fit.f90 -- fitting for circe
<Copyleft notice 30a>

module fit_routines
  use kinds

  implicit none
  private

  <circe1_fit.f90: public 95b>

  contains
  <circe1_fit.f90: subroutines 95c>
end module fit_routines

program fit
  use kinds
  use fit_routines
```

```

implicit none

integer :: i, rcode
<Declare NPARAM 94a>
<Declare parameters 94b>
<Declare arguments 94c>

<Initialize parameters for circe1_fit.f90 95a>
call mninit (5, 6, 7)
<Load parameters 94d>
call mnseti ('CIRCE: fit version 1      ')
argv(1) = 1
call mnexcm (fct, 'SET PRINTOUT      ', argv, 1, rcode, 0d0)
argv(1) = 1
call mnexcm (fct, 'CALL FCT          ', argv, 1, rcode, 0d0)
call mnexcm (fct, 'MIGRAD            ', argv, 0, rcode, 0d0)
call mnexcm (fct, 'MINOS             ', argv, 0, rcode, 0d0)
argv(1) = 3
call mnexcm (fct, 'CALL FCT          ', argv, 1, rcode, 0d0)
call mnexcm (fct, 'STOP              ', argv, 0, rcode, 0d0)

end program fit

```

Defines:
fit, used in chunks 94d, 111, and 115d.
Uses circe 31b and fct 95c 111c.

94a *<Declare NPARAM 94a>*≡
integer, parameter :: NPARAM = 6

Defines:
NPARAM, used in chunks 94 and 101b.

94b *<Declare parameters 94b>*≡
integer, dimension(NPARAM) :: pnum
character(len=10), dimension(NPARAM) :: pname
real(kind=double), dimension(NPARAM) :: pstart, pstep

Uses NPARAM 94a.

94c *<Declare arguments 94c>*≡
integer, parameter :: ARGV = 10
real(kind=double), dimension(ARGV) :: argv

94d *<Load parameters 94d>*≡
do i = 1, NPARAM
call mnparm (pnum(i), pname(i), pstart(i), pstep (i), 0d0, 0d0, rcode)
if (rcode .ne. 0) then
print *, "fit: MINUIT won't accept parameter ", pnum(i)
stop
endif
end do

Uses NPARAM 94a and fit 93b.

```

95a  <Initialize parameters for circe1_fit.f90 95a>≡
      data pnum / 1, 2, 3, 4, 5, 6 /
      data pname / '1_e', 'x_e', '1-x_e', '1_g', 'x_g', '1-x_g' /
      data pstart / -1.00, 20.00, 0.20, -1.00, 0.20, 20.00 /
      data pstep / 0.01, 0.01, 0.01, 0.01, 0.01, 0.01 /

95b  <circe1_fit.f90: public 95b>≡
      public :: fct
      Uses fct 95c 111c.

95c  <circe1_fit.f90: subroutines 95c>≡
      subroutine fct (nx, df, f, a, mode, g)
        integer :: nx, mode
        real(kind=double) :: f, g
        real(kind=double), dimension(:) :: df, a
        <Local variables for fct (v1) 95f>
        if (mode .eq. 1) then
          <Read input data (v1) 95d>
        else if (mode .eq. 2) then
          <Calculate  $\nabla f$  99a>
        end if
        <Calculate f (v1) 99b>
        end if
        if (mode .eq. 3) then
          <Write output (v1) 101a>
        end if
      end subroutine fct

Defines:
      fct, used in chunks 93b, 95b, 110, 111, and 116a.

95d  <Read input data (v1) 95d>≡
      <Read data from file 95e>
      <Fixup errors 97b>
      <Normalize 97e>

95e  <Read data from file 95e>≡
      call gethst ('ee', NDATA, xee, fee, dfec, see, tee, pwr)
      call gethst ('eg', NDATA, xeg, feg, dfeg, seg, teg, pwr)
      call gethst ('ge', NDATA, xge, fge, dfge, sge, tge, pwr)
      call gethst ('gg', NDATA, xgg, fgg, dfgg, sgg, tgg, pwr)
      Uses gethst 96b.

95f  <Local variables for fct (v1) 95f>≡
      integer, parameter :: NDATA = 20
      real(kind=double) :: see, tee, dtee
      real(kind=double) :: seg, teg, dteg
      real(kind=double) :: sge, tge, dtge
      real(kind=double) :: sgg, tgg, dtgg
      real(kind=double), dimension(2,0:NDATA+1,0:NDATA+1) :: xee, xeg, &
        xge, xgg
      real(kind=double), dimension(0:NDATA+1,0:NDATA+1) :: fee, dfec, &

```

```

        feg, dfeg, fge, dfge, fgg, dfgg
        real(kind=double) :: pwr
96a  <circe1_fit.f90: public 95b>+≡
        public :: gethst
        Uses gethst 96b.
96b  <circe1_fit.f90: subroutines 95c>+≡
        subroutine gethst (tag, ndata, x, f, df, s, t, pwr)
            character(len=2) :: tag
            integer :: ndata
            real(kind=double) :: s, t, pwr
            real(kind=double), dimension(2,0:ndata+1,0:ndata+1) :: x
            real(kind=double), dimension(0:ndata+1,0:ndata+1) :: f, df
            integer :: i, j
            open (10, file = 'lumidiff-'//tag//'.dat')
            read (10, *) pwr
            s = 0d0
            <Read continuum, summing in s 96c>
            t = s
            <Read single  $\delta$ , summing in t 96d>
            <Read double  $\delta$ , summing in t 97a>
            close (10)
        end subroutine gethst

```

Defines:

gethst, used in chunks 95e and 96a.

```

96c  <Read continuum, summing in s 96c>≡
        do i = 1, ndata
            do j = 1, ndata
                read (10, *) x(1,i,j), x(2,i,j), f(i,j), df(i,j)
                s = s + f(i,j)
            end do
        end do
96d  <Read single  $\delta$ , summing in t 96d>≡
        do i = 1, ndata
            read (10, *) x(1,i,0), f(i,0), df(i,0), &
                f(i,ndata+1), df(i,ndata+1)
            x(1,i,ndata+1) = x(1,i,0)
            t = t + f(i,0) + f(i,ndata+1)
        end do
96e  <Read single  $\delta$ , summing in t 96d>+≡
        do i = 1, ndata
            read (10, *) x(2,0,i), f(0,i), df(0,i), &
                f(ndata+1,i), df(ndata+1,i)
            x(2,ndata+1,i) = x(2,0,i)
            t = t + f(0,i) + f(ndata+1,i)
        end do

```

97a *<Read double δ , summing in t 97a>*≡

```

      read (10, *) f(0,0), df(0,0), f(0,ndata+1), df(0,ndata+1)
      t = t + f(0,0) + f(0,ndata+1)
      read (10, *) f(ndata+1,0), df(ndata+1,0), &
                f(ndata+1,ndata+1), df(ndata+1,ndata+1)
      t = t + f(ndata+1,0) + f(ndata+1,ndata+1)

```

Guinea-Pig does not provide the full error. A Monte Carlo study shows that it is a reasonable approximation to rescale the bin error by suitable factors. These factors are different for each distribution and the factors for the δ -pieces are bigger than those for the continuum parts. The follows factors are for the **slow** parameter set.

97b *<Fixup errors 97b>*≡

```

      call fixerr (NDATA, dfec, 20d0, 30d0, 40d0)
      call fixerr (NDATA, dfeg, 15d0, 20d0, 0d0)
      call fixerr (NDATA, dfge, 15d0, 20d0, 0d0)
      call fixerr (NDATA, dfgg, 10d0, 0d0, 0d0)

```

Uses **fixerr** 97d.

97c *<circe1_fit.f90: public 95b>*+≡

```

      public :: fixerr

```

Uses **fixerr** 97d.

97d *<circe1_fit.f90: subroutines 95c>*+≡

```

      subroutine fixerr (ndata, df, c, sd, dd)
      integer :: ndata
      real(kind=double) :: c, sd, dd
      real(kind=double), dimension(0:ndata+1,0:ndata+1) :: df
      integer :: i, j
      do i = 1, NDATA
        do j = 1, NDATA
          df(i,j) = c * df(i,j)
        end do
      end do
      do i = 1, NDATA
        df(0,i) = sd * df(0,i)
        df(i,0) = sd * df(i,0)
        df(ndata+1,i) = sd * df(ndata+1,i)
        df(i,ndata+1) = sd * df(i,ndata+1)
      end do
      df(0,0) = dd * df(0,0)
      df(ndata+1,0) = dd * df(ndata+1,0)
      df(0,ndata+1) = dd * df(0,ndata+1)
      df(ndata+1,ndata+1) = dd * df(ndata+1,ndata+1)
      end subroutine fixerr

```

Defines:

fixerr, used in chunk 97.

The error on the integrated luminosity is obtained from adding the error in channels in quadrature.

97e *<Normalize 97e>*≡

```

dtee = sumsqu (NDATA, dfce)
dteg = sumsqu (NDATA, dfeg)
dtge = sumsqu (NDATA, dfge)
dtgg = sumsqu (NDATA, dfgg)

```

Uses sumsqu 98b.

```

98a <circe1_fit.f90: public 95b>+≡
    public :: sumsqu

```

Uses sumsqu 98b.

```

98b <circe1_fit.f90: subroutines 95c>+≡
    function sumsqu (ndata, f)
        integer :: ndata
        real(kind=double) :: sumsqu
        real(kind=double), dimension(0:ndata+1,0:ndata+1) :: f
        integer :: i, j
        real(kind=double) :: s2
        s2 = 0
        do i = 0, NDATA+1
            do j = 0, NDATA+1
                s2 = s2 + f(i,j)*f(i,j)
            end do
        end do
        sumsqu = sqrt (s2)
    end function sumsqu

```

Defines:

sumsqu, used in chunks 97e and 98a.

```

98c <Normalize 97e>+≡
    call scale (NDATA, 1d0/tee, fee)
    call scale (NDATA, 1d0/tee, dfce)
    call scale (NDATA, 1d0/tee, feg)
    call scale (NDATA, 1d0/tee, dfeg)
    call scale (NDATA, 1d0/tee, fge)
    call scale (NDATA, 1d0/tee, dfge)
    call scale (NDATA, 1d0/tee, fgg)
    call scale (NDATA, 1d0/tee, dfgg)

```

Uses scale 98e.

```

98d <circe1_fit.f90: public 95b>+≡
    public :: scale

```

Uses scale 98e.

```

98e <circe1_fit.f90: subroutines 95c>+≡
    subroutine scale (ndata, s, f)
        integer :: ndata
        real(kind=double) :: s
        real(kind=double), dimension(0:ndata+1,0:ndata+1) :: f
        integer :: i, j
        do i = 0, NDATA+1
            do j = 0, NDATA+1

```

```

        f(i,j) = s * f(i,j)
    end do
end do
end subroutine scale

```

Defines:

`scale`, used in chunk 98.

99a $\langle \text{Calculate } \nabla f \text{ 99a} \rangle \equiv$

```

    print *, "ERROR: $\nabla f$ n.a."
    stop

```

Log-likelihood won't fly, because we can't normalize the likelihood function for an unbounded parameter range. Let's use good ole least-squares instead.

99b $\langle \text{Calculate } f \text{ (v1) 99b} \rangle \equiv$

```

    f = 0d0
    do i = 1, NDATA
        do j = 1, NDATA
            if (dfee(i,j) .gt. 0d0) then
                f = f + ((phie(xee(1,i,j),a) * phie(xee(2,i,j),a) &
                    - fee(i,j)) / dfee(i,j))**2
            end if
            if (dfeg(i,j) .gt. 0d0) then
                f = f + ((phie(xeg(1,i,j),a) * phig(xeg(2,i,j),a) &
                    - feg(i,j)) / dfeg(i,j))**2
            end if
            if (dfge(i,j) .gt. 0d0) then
                f = f + ((phig(xge(1,i,j),a) * phie(xge(2,i,j),a) &
                    - fge(i,j)) / dfge(i,j))**2
            end if
            if (dfgg(i,j) .gt. 0d0) then
                f = f + ((phig(xgg(1,i,j),a) * phig(xgg(2,i,j),a) &
                    - fgg(i,j)) / dfgg(i,j))**2
            end if
        end do
    end do

```

Uses `phie` 100e and `phig` 100g.

99c $\langle \text{Local variables for fct (v1) 95f} \rangle \equiv$

```

    integer :: i, j
    real(kind=double) :: delta

```

99d $\langle \text{Calculate } f \text{ (v1) 99b} \rangle \equiv$

```

    if ((a(2) .le. -1d0) .or. (a(3) .le. -1d0/pwr)) then
        print *, "warning: discarding out-of-range a2/3: ", a(2), a(3)
         $\langle \text{Give up on } f \text{ 100a} \rangle$ 
    else
        delta = 1d0 - exp(a(1)) * beta(a(2)+1d0,a(3)+1d0/pwr) * dble(NDATA) / pwr
        if (delta .lt. 0d0) then
            print *, "warning: delta forced to 0 from ", delta
            delta = 0d0
        end if
    end if

```

Uses beta 105.

100a $\langle \text{Give up on } f \text{ 100a} \rangle \equiv$
 $f = 1d100$

100b $\langle \text{Calculate } f(v1) \text{ 99b} \rangle + \equiv$
do i = 1, NDATA
 if (dfee(ndata+1,i) .gt. 0d0) then
 f = f + ((delta*phie(xee(2,ndata+1,i),a) &
 - fee(ndata+1,i)) / dfee(ndata+1,i))**2
 end if
 if (dfeg(ndata+1,i) .gt. 0d0) then
 f = f + ((delta*phig(xeg(2,ndata+1,i),a) &
 - feg(ndata+1,i)) / dfeg(ndata+1,i))**2
 end if
 if (dfee(i,ndata+1) .gt. 0d0) then
 f = f + ((delta*phie(xee(1,i,ndata+1),a) &
 - fee(i,ndata+1)) / dfee(i,ndata+1))**2
 end if
 if (dfge(i,ndata+1) .gt. 0d0) then
 f = f + ((delta*phig(xge(1,i,ndata+1),a) &
 - fge(i,ndata+1)) / dfge(i,ndata+1))**2
 end if
end do

Uses phie 100e and phig 100g.

100c $\langle \text{Calculate } f(v1) \text{ 99b} \rangle + \equiv$
 if (dfee(ndata+1,ndata+1) .gt. 0d0) then
 f = f + ((delta*delta &
 - fee(ndata+1,ndata+1)) / dfee(ndata+1,ndata+1))**2
 end if

100d $\langle \text{circe1_fit.f90: public 95b} \rangle + \equiv$
public :: phie

Uses phie 100e.

100e $\langle \text{circe1_fit.f90: subroutines 95c} \rangle + \equiv$
function phie (x, a)
 real(kind=double) :: x, phie
 real(kind=double), dimension(6) :: a
 phie = exp (a(1) + a(2)*log(x) + a(3)*log(1d0-x))
end function phie

Defines:

 phie, used in chunks 99, 100, and 103b.

100f $\langle \text{circe1_fit.f90: public 95b} \rangle + \equiv$
public :: phig

Uses phig 100g.

100g $\langle \text{circe1_fit.f90: subroutines 95c} \rangle + \equiv$
function phig (x, a)
 real(kind=double) :: x, phig

```

      real(kind=double), dimension(6) :: a
      phig = exp (a(4) + a(5)*log(x) + a(6)*log(1d0-x))
end function phig

```

Defines:

phig, used in chunks 99, 100, and 103b.

101a *<Write output (v1) 101a>*≡

```

      a1(1) = exp(a(1)) * dble(NDATA) / pwr
      a1(2) = a(2)
      a1(3) = a(3) - 1d0 + 1d0/pwr
      a1(4) = exp(a(4)) * dble(NDATA) / pwr
      a1(5) = a(5) - 1d0 + 1d0/pwr
      a1(6) = a(6)
      open (10, file = 'Parameters')
      write (10, 1000) REV, tee / 1D32
      write (10, 1001) REV, &
        1d0 - a1(1) * beta(a1(2)+1d0,a1(3)+1d0), &
        a1(1), a1(2), a1(3), a1(4), a1(5), a1(6), &
        a1(4) * beta(a1(5)+1d0,a1(6)+1d0)
      1000 format ('      data xa5lum(@ENERGY@,@ACC@,', I2, ') / ', E12.5, ' /')
      1001 format ('      data (xa5(i,@ENERGY@,@ACC@,', I2, '),i=0,7) /', /, &
        '      $ ', 4(E12.5,', '), /, &
        '      $ ', 3(E12.5,', '), E12.5, ' /')
      close (10)

```

Uses beta 105.

101b *<Local variables for fct (v1) 95f>*+≡

```

      <Declare NPARAM 94a>
      real(kind=double), dimension(NPARAM) :: a1
      integer, parameter :: REV = 1

```

Uses NPARAM 94a.

The average elektron energy in the continuum can be calculated analytically:

$$\begin{aligned}
 \langle E_{e^\pm} \rangle_{\text{cont}} &= E_{\text{beam}} \langle x_{e^\pm} \rangle_{\text{cont}} = E_{\text{beam}} \frac{\int dx x^{a_2} (1-x)^{a_3} x}{B(a_2, a_3)} \\
 &= E_{\text{beam}} \frac{B(a_2 + 1, a_3)}{B(a_2, a_3)} = E_{\text{beam}} \frac{a_2 + 1}{a_2 + a_3 + 2} \quad (40)
 \end{aligned}$$

101c *<Write output (v1) 101a>*+≡

```

      delta = 1d0 - a1(1) * beta(a1(2)+1d0,a1(3)+1d0)
      print *, '< x_e > = ', delta + (1d0-delta)*(a1(2)+1d0)/(a1(2)+a1(3)+2d0)

```

Uses beta 105.

similarly:

$$\langle E_\gamma \rangle = E_{\text{beam}} \frac{a_5 + 1}{a_5 + a_6 + 2} \quad (41)$$

101d *<Write output (v1) 101a>*+≡

```

      print *, '< x_g > = ', (a1(5)+1d0)/(a1(5)+a1(6)+2d0)

```

Count the degrees of freedom in ndof:

```
102a <Write output (v1) 101a>+≡
      ndof = 0
      do i = 0, ndata+1
        do j = 0, ndata+1
          if (dfec(i,j) .gt. 0d0) ndof = ndof + 1
          if (dfeg(i,j) .gt. 0d0) ndof = ndof + 1
          if (dfge(i,j) .gt. 0d0) ndof = ndof + 1
          if (dfgg(i,j) .gt. 0d0) ndof = ndof + 1
        end do
      end do
      print *, 'CHI2 = ', f / ndof
```

```
102b <Local variables for fct (v1) 95f>+≡
      integer :: ndof
```

The error on the luminosity is just the (possibly rescaled) counting error:

```
102c <Write output (v1) 101a>+≡
      open (10, file = 'Errors.tex')
      write (10, 1099) tee / 1d32, dtee / 1d32, dtee / 1d32
      1099 format ('$ ', F8.2, ' _{-', F4.2, ' }^{+', F4.2, ' }$')
```

After retrieving the error from MINUIT, we have to take care of the mapping of the parameters

$$a'_{1/4} = e^{a_{1/4}} B(a_{2/5} + 1, a_{3/6} + 1) N_{\text{bins}} \eta^{-1} \implies \delta a'_{1/4} = a'_{1/4} \delta a_{1/4} \quad (42)$$

ignoring the errors in the integral (i.e. the Beta function).

```
102d <Write output (v1) 101a>+≡
      call mnerrs (1, eplus, eminus, epara, corr)
      ab = a1(1) * beta(a1(2)+1d0, a1(3)+1d0)
      write (10, 1100) ab, abs (ab*eminus), abs (ab*eplus)
      1100 format ('$ ', F8.4, ' _{-', F6.4, ' }^{+', F6.4, ' }$')
```

Uses beta 105.

```
102e <Local variables for fct (v1) 95f>+≡
      real(kind=double) :: ab
```

The other mappings are even more trivial:

$$a'_{2/6} = a_{2/6} - 1 + \eta^{-1} \implies \delta a'_{2/6} = \delta a_{2/6} \quad a'_{3/5} = a_{3/5} - 1 + \eta^{-1} \implies \delta a'_{3/5} = \delta a_{3/5} \quad (43)$$

```
102f <Write output (v1) 101a>+≡
      do i = 2, 3
        call mnerrs (i, eplus, eminus, epara, corr)
        write (10, 1100) a1(i), abs (eminus), abs (eplus)
      end do
      call mnerrs (4, eplus, eminus, epara, corr)
      ab = a1(4) * beta(a1(5)+1d0, a1(6)+1d0)
      write (10, 1100) ab, abs (ab*eminus), abs (ab*eplus)
      do i = 5, 6
        call mnerrs (i, eplus, eminus, epara, corr)
```

```

        write (10, 1100) a1(i), abs (eminus), abs (eplus)
    end do
    close (10)
    Uses beta 105.
103a  <Local variables for fct (v1) 95f>+≡
      real(kind=double) :: eplus, eminus, epara, corr
      integer :: n
103b  <Write output (v1) 101a>+≡
      do n = 1, 10
        call pslice ('ee','x',n,NDATA,xee,fee,dfee,phie,phie,a)
        call pslice ('eg','x',n,NDATA,xeg,feg,dfeg,phie,phig,a)
        call pslice ('ge','x',n,NDATA,xge,fge,dfge,phig,phie,a)
        call pslice ('gg','x',n,NDATA,xgg,fgg,dfgg,phig,phig,a)
        call pslice ('ee','y',n,NDATA,xee,fee,dfee,phie,phie,a)
        call pslice ('eg','y',n,NDATA,xeg,feg,dfeg,phie,phig,a)
        call pslice ('ge','y',n,NDATA,xge,fge,dfge,phig,phie,a)
        call pslice ('gg','y',n,NDATA,xgg,fgg,dfgg,phig,phig,a)
      end do
      call pslice ('ee','x',21,NDATA,xee,fee,dfee,phie,phie,a)
      call pslice ('eg','x',21,NDATA,xeg,feg,dfeg,phie,phig,a)
      call pslice ('ee','y',21,NDATA,xee,fee,dfee,phie,phie,a)
      call pslice ('ge','y',21,NDATA,xge,fge,dfge,phig,phie,a)
    Uses phie 100e, phig 100g, and pslice 103e.
    UNIX Fortran compiler want backslashes escaped:
103c  <Write output (v1) 101a>+≡
      open (10, file = 'Slices.mp4')
      write (10,*) "picture eslice[], gslice[];"
      do n = 1, NDATA
        write (10,*) 'eslice[', n, ']' := ', &
          'btex $x_{e^{\backslash pm}} = ', xee(1,n,1), '$ etex;'
        write (10,*) 'gslice[', n, ']' := ', &
          'btex $x_{\backslash gamma} = ', xgg(1,n,1), '$ etex;'
      end do
      close (10)
103d  <circe1_fit.f90: public 95b>+≡
      public :: pslice
    Uses pslice 103e.
103e  <circe1_fit.f90: subroutines 95c>+≡
      subroutine pslice (pp, xy, n, ndata, x, f, df, phi1, phi2, a)
        character(len=2) :: pp
        character(len=1) :: xy
        integer :: n, ndata
        real(kind=double), dimension(2,0:ndata+1,0:ndata+1) :: x
        real(kind=double), dimension(0:ndata+1,0:ndata+1) :: f, df
        real(kind=double), dimension(6) :: a
        real(kind=double) :: z
        real(kind=double) :: phi1, phi2, d, delta, pwr

```

```

external phi1, phi2
integer :: i
character(len=2) digits
write (digits, '(I2.2)') n
open (10, file = 'lumidiff-//pp//xy//digits//'.dat')
open (11, file = 'lumidiff-//pp//xy//digits//'.fit')
open (12, file = 'lumidiff-//pp//xy//digits//'.chi')
if (n .eq. ndata+1) then
    pwr = 5d0
    delta = 1d0 - exp(a(1))*beta(a(2)+1d0,a(3)+1d0/pwr) &
        * dble(NDATA) / pwr
else
    delta = 0
end if
if (xy .eq. 'x') then
    do i = 1, ndata
        if (df(n,i) .gt. 0d0) then
            if (pp(2:2) .eq. 'g') then
                z = x(2,n,i)
            else
                z = 1d0 - x(2,n,i)
            endif
            if (n .eq. ndata+1) then
                d = delta*phi2(x(2,n,i),a)
            else
                d = phi1(x(1,n,i),a)*phi2(x(2,n,i),a)
            endif
            write (10,*) z, f(n,i), df(n,i)
            write (11,*) z, d
            write (12,*) z, (f(n,i) - d) / df(n,i)
        endif
    end do
else if (xy .eq. 'y') then
    do i = 1, ndata
        if (df(i,n) .gt. 0d0) then
            if (pp(1:1) .eq. 'g') then
                z = x(1,i,n)
            else
                z = 1d0 - x(1,i,n)
            endif
            if (n .eq. ndata+1) then
                d = phi1(x(1,i,n),a)*delta
            else
                d = phi1(x(1,i,n),a)*phi2(x(2,i,n),a)
            endif
            write (10,*) z, f(i,n), df(i,n)
            write (11,*) z, d
            write (12,*) z, (f(i,n) - d) / df(i,n)
        endif
    end do
end do

```

```

endif
close (10)
close (11)
close (12)
end subroutine pslice

```

Defines:

pslice, used in chunk 103.

Uses beta 105.

```

105 (circe1_fit.f90: subroutines 95c)+≡
function beta (a, b)
real(kind=double) :: a, b, beta
beta = exp (dlgamma(a) + dlgama(b) - dlgama(a+b))
contains
function dlgama (x)
real(kind=double) :: dlgama
real(kind=double), dimension(7) :: p1, q1, p2, q2, p3, q3
real(kind=double), dimension(5) :: c, xl
real(kind=double) :: x, y, zero, one, two, half, ap, aq
integer :: i
data ZERO /0.0D0/, ONE /1.0D0/, TWO /2.0D0/, HALF /0.5D0/
data XL /0.0D0,0.5D0,1.5D0,4.0D0,12.0D0/
data p1 /+3.8428736567460D+0, +5.2706893753010D+1, &
+5.5584045723515D+1, -2.1513513573726D+2, &
-2.4587261722292D+2, -5.7500893603041D+1, &
-2.3359098949513D+0/
data q1 /+1.0000000000000D+0, +3.3733047907071D+1, &
+1.9387784034377D+2, +3.0882954973424D+2, &
+1.5006839064891D+2, +2.0106851344334D+1, &
+4.5717420282503D-1/
data p2 /+4.8740201396839D+0, +2.4884525168574D+2, &
+2.1797366058896D+3, +3.7975124011525D+3, &
-1.9778070769842D+3, -3.6929834005591D+3, &
-5.6017773537804D+2/
data q2 /+1.0000000000000D+0, +9.5099917418209D+1, &
+1.5612045277929D+3, +7.2340087928948D+3, &
+1.0459576594059D+4, +4.1699415153200D+3, &
+2.7678583623804D+2/
data p3 /-6.8806240094594D+3, -4.3069969819571D+5, &
-4.7504594653440D+6, -2.9423445930322D+6, &
+3.6321804931543D+7, -3.3567782814546D+6, &
-2.4804369488286D+7/
data q3 /+1.0000000000000D+0, -1.4216829839651D+3, &
-1.5552890280854D+5, -3.4152517108011D+6, &
-2.0969623255804D+7, -3.4544175093344D+7, &
-9.1605582863713D+6/
data c / 1.1224921356561D-1, 7.9591692961204D-2, &
-1.7087794611020D-3, 9.1893853320467D-1, &
1.3469905627879D+0/
if (x .le. xl(1)) then

```

```

        print *, 'ERROR: DLGAMA non positive argument: ', X
        dlgama = zero
    end if
    if (x .le. xl(2)) then
        y = x + one
        ap = p1(1)
        aq = q1(1)
        do i = 2, 7
            ap = p1(i) + y * ap
            aq = q1(i) + y * aq
        end do
        y = - log(x) + x * ap / aq
    else if (x .le. xl(3)) then
        ap = p1(1)
        aq = q1(1)
        do i = 2, 7
            ap = p1(i) + x * ap
            aq = q1(i) + x * aq
        end do
        y = (x - one) * ap / aq
    else if (x .le. xl(4)) then
        ap = p2(1)
        aq = q2(1)
        do i = 2, 7
            ap = p2(i) + x * ap
            aq = q2(i) + x * aq
        end do
        y = (x-two) * ap / aq
    else if (x .le. xl(5)) then
        ap = p3(1)
        aq = q3(1)
        do i = 2, 7
            ap = p3(i) + x * ap
            aq = q3(i) + x * aq
        end do
        y = ap / aq
    else
        y = one / x**2
        y = (x-half) * log(x) - x + c(4) + &
            (c(1) + y * (c(2) + y * c(3))) / ((c(5) + y) * x)
    end if
    dlgama = y
end function dlgama
end function beta

```

Defines:

beta, used in chunks [59b](#), [61e](#), [65c](#), [70b](#), [71f](#), [75a](#), [99d](#), [101–103](#), and [108b](#).

```

106 <circe1_fit.sh 106>≡
    #! /bin/sh

```

```

# mode=${2-slow}
mode=${2-fast}
root='pwd'
indir=${root}/${3-input}
tmpdir=${root}/tmp
outdir=${root}/output
acc="${1-sband350 sband500 sband800 sband1000 sband1600
      tesla350 tesla500 tesla800 tesla1000 tesla1600
      tesla350-low tesla500-low tesla800-low tesla1000-low tesla1600-low
      xband350 xband500 xband800 xband1000 xband1600}"

107a <circe1_fit.sh 106>+=
    xmkdir () {
        for d in "$@"; do
            mkdir $d 2>/dev/null || true
        done
    }
    rm -fr ${tmpdir}
    xmkdir ${outdir} ${tmpdir}

107b <circe1_fit.sh 106>+=
    cd ${tmpdir}
    cat /dev/null >${outdir}/Params.f90
    for a in $acc; do
        case "$a" in
            *1600*) energy=TEV16;;
            *1000*) energy=TEV1;;
            *800*) energy=GEV800;;
            *500*) energy=GEV500;;
            *3[56]0*) energy=GEV350;;
            *170*) energy=GEV170;;
            *90*) energy=GEV090;;
            *) energy=GEV500;;
        esac
        cp ${indir}/${a}_${mode}/lumidiff-???.dat .
        ${root}/circe1_fit.bin
        rm -fr ${outdir}/${a}_${mode}
        mkdir ${outdir}/${a}_${mode}
        cp Slices.mp4 ${outdir}
        cp Errors.tex lumidiff-??x[0-9][0-9].??? ${outdir}/${a}_${mode}
        sed -e "s/@ENERGY@/$energy/g" \
            -e "s/@ACC@/'echo $a | tr a-z A-Z | tr -cd A-Z'/g" Parameters \
            >>${outdir}/Params.f90
    done
    cd ${root}
    rm -fr ${tmpdir}

107c <circe1_fit.sh 106>+=
    cat >${outdir}/Params.tex <<'END'
    \begin{table}
    \begin{center}

```

```

\renewcommand{\arraystretch}{1.3}
\begin{tabular}{|c||c|c|c|c|}\hline
& \texttt{SBAND} & \texttt{TESLA} & \texttt{TESLA'} & \texttt{XBAND} \\
\\ \hline \hline
END
Uses SBAND 13a, TESLA 13a, and XBAND 13a.
108a <circe1_fit.sh 106>+=
line () {
for a in $acc; do
case $a in
*350* | *800* | *1000* | *1600*)
;;
*) echo -n ' & '
sed -n $1p ${outdir}/${a}_${mode}/Errors.tex
;;
esac
done
echo '\\\hline'
}
(echo '$\mathcal{L}/\text{fb}^{-1}\upsilon^{-1}$'; line 1
echo '$\int d_{\text{e}^{\text{pm}}}$'; line 2
echo '$x_{\text{e}^{\text{pm}}}^{\alpha}$'; line 3
echo '$(1-x_{\text{e}^{\text{pm}}})^{\alpha}$'; line 4
echo '$\int d_{\gamma}$'; line 5
echo '$x_{\gamma}^{\alpha}$'; line 6
echo '$(1-x_{\gamma})^{\alpha}$'; line 7
) >>${outdir}/Params.tex
108b <circe1_fit.sh 106>+=
cat >>${outdir}/Params.tex <<'END'
\end{tabular}
\end{center}
\caption{\label{tab:param}%
Version 1, revision 1997 04 16 of the beam spectra at 500 GeV.
The rows correspond to the luminosity per effective year, the
integral over the continuum and the powers in the factorized Beta
distributions~(\ref{eq:beta}).}
\end{table}
END
Uses beta 105.
108c <circe1_fit.sh 106>+=
cat >>${outdir}/Params.tex <<'END'
\begin{table}
\begin{center}
\renewcommand{\arraystretch}{1.3}
\begin{tabular}{|c||c|c|c|c|}\hline
& \texttt{SBAND} & \texttt{TESLA} & \texttt{TESLA'} & \texttt{XBAND} \\
\\ \hline \hline
END
Uses SBAND 13a, TESLA 13a, and XBAND 13a.

```

```

109a <circe1_fit.sh 106>+=
line () {
    for a in $acc; do
        case $a in
            *1000*)
                echo -n ' & '
                sed -n $1p ${outdir}/${a}_${mode}/Errors.tex
                ;;
            esac
        done
        echo '\\\\hline'
    }
(echo '$\mathcal{L}/\text{fb}^{-1}\upsilon^{-1}$'; line 1
echo '$\int d_{e\pm}$'; line 2
echo '$x_{e\pm}^{\alpha}$'; line 3
echo '$(1-x_{e\pm})^{\alpha}$'; line 4
echo '$\int d_{\gamma}$'; line 5
echo '$x_{\gamma}^{\alpha}$'; line 6
echo '$(1-x_{\gamma})^{\alpha}$'; line 7
) >>${outdir}/Params.tex

109b <circe1_fit.sh 106>+=
cat >>${outdir}/Params.tex <<'END'
    \end{tabular}
    \end{center}
    \caption{\label{tab:param/TeV}%
        Version 1, revision 1997 04 17 of the beam spectra at 1 TeV.}
\end{table}
END

109c <circe1_fit.sh 106>+=
cat >>${outdir}/Params.tex <<'END'
\begin{table}
    \begin{center}
        \renewcommand{\arraystretch}{1.3}
        \begin{tabular}{|c||c|c|c|c|}\hline
            & 350 GeV & 500 GeV & 800 GeV & 1600 GeV \\
            \\\hline\hline
        \end{tabular}
    \end{center}
END

109d <circe1_fit.sh 106>+=
line () {
    for a in $acc; do
        case $a in
            tesla*-low)
                ;;
            tesla1000)
                ;;
            tesla*)
                echo -n ' & '
                sed -n $1p ${outdir}/${a}_${mode}/Errors.tex

```

```

        ;;
    esac
done
echo '\\\\hline'
}
(echo '$\mathcal{L}/\text{fb}^{-1}\text{\\upsilon}^{-1}$'; line 1
echo '$\int d_{\text{e}^{\text{pm}}}$'; line 2
echo '$x_{\text{e}^{\text{pm}}}^{\alpha}$'; line 3
echo '$(1-x_{\text{e}^{\text{pm}}})^{\alpha}$'; line 4
echo '$\int d_{\gamma}$'; line 5
echo '$x_{\gamma}^{\alpha}$'; line 6
echo '$(1-x_{\gamma})^{\alpha}$'; line 7
) >>${outdir}/Params.tex
110a <circe1_fit.sh 106>+=
    cat >>${outdir}/Params.tex <<'END'
        \end{tabular}
        \end{center}
        \caption{\label{tab:param/Tesla}%
            Version 1, revision 1997 04 17 of the beam spectra for TESLA.}
        \end{table}
    END
    exit 0
Uses TESLA 13a.

```

7.2 Experimental

7.2.1 Quasi One Dimensional

```

110b <circe1_minuit1.f90 110b>≡
    ! circe1_minuit1.f90 -- fitting for circe
    <Copyleft notice 30a>
Uses circe 31b.

```

We're utilizing the familiar “MINUIT” package [15].

```

110c <circe1_minuit1.f90 110b>+=
    <Minuit1 module 110d>
    <Minuit1 main program 111a>

```

```

110d <Minuit1 module 110d>≡
    module minuit1
        use kinds

        implicit none

        public :: fct
        public :: phi

        contains

```

⟨Function to minimize 111c⟩

⟨Function phi1 113⟩
end module minuit1

Defines:

minuit1, used in chunk 111a.

Uses fct 95c 111c and phi 113 116b.

111a *⟨Minuit1 main program 111a⟩*≡

```
program fit
  use kinds
  use minuit1

  implicit none

  call minuit (fct, 0d0)
end program fit
```

Uses fct 95c 111c, fit 93b, and minuit1 110d.

111b *⟨Minuit2 main program 111b⟩*≡

```
program fit
  use kinds
  use minuit2

  implicit none

  call minuit (fct, 0d0)
end program fit
```

Uses fct 95c 111c, fit 93b, and minuit2 116a.

111c *⟨Function to minimize 111c⟩*≡

```
subroutine fct (nx, df, f, a, mode, g)
  integer, intent(in) :: nx, mode
  real(kind=double) :: f, g
  real(kind=double), dimension(:) :: df, a
  ⟨Local variables for fct 112b⟩
  if (mode .eq. 1) then
    ⟨Read input data 112a⟩
  else if (mode .eq. 2) then
    ⟨Calculate ∇f 99a⟩
  end if
  ⟨Calculate f 112c⟩
  if (mode .eq. 3) then
    ⟨Write output 112d⟩
  end if
end subroutine fct
```

Defines:

fct, used in chunks 93b, 95b, 110, 111, and 116a.

```

112a  <Read input data 112a>≡
      open (10, file = 'minuit.data')
      do i = 1, NDATA
        do j = 1, NDATA
          read (10, *) xi(1,i,j), xi(2,i,j), fi(i,j), dfi(i,j)
          fi(i,j) = fi(i,j)/1d30
          dfi(i,j) = dfi(i,j)/1d30
        end do
      end do
      close (10)

112b  <Local variables for fct 112b>≡
      integer, parameter :: NDATA = 20
      real(kind=double) :: chi, chi2
      real(kind=double), dimension(2,NDATA,NDATA) :: xi
      real(kind=double), dimension(NDATA,NDATA) :: fi, dfi
      integer :: i, j, n

112c  <Calculate f 112c>≡
      f = 0d0
      do i = 1, NDATA
        do j = 1, NDATA
          if (dfi(i,j).gt.0d0) then
            f = f + ((phi(xi(1,i,j),xi(2,i,j),a) &
              - fi(i,j)) / dfi(i,j))**2
          end if
        end do
      end do
      Uses phi 113 116b.

112d  <Write output 112d>≡
      chi2 = 0d0
      n = 0
      open (10, file = 'minuit.fit')
      do i = 1, NDATA
        do j = 1, NDATA
          if (dfi(i,j).gt.0d0) then
            chi = (phi(xi(1,i,j),xi(2,i,j),a)-fi(i,j))/dfi(i,j)
            write (10,*) xi(1,i,j), xi(2,i,j), &
              1d30 * phi(xi(1,i,j),xi(2,i,j),a), &
              1d30 * fi(i,j), &
              chi
            chi2 = chi2 + chi**2
            n = n + 1
          else
            write (10,*) xi(1,i,j), xi(2,i,j), &
              1d30 * phi(xi(1,i,j),xi(2,i,j),a), &
              1d30 * fi(i,j)
          end if
        end do
      end do

```

```

        close (10)
        print *, 'CHI2 = ', chi2/n
    Uses phi 113 116b.
113  (Function phi1 113)≡
    function phi (e1, e2, a)
        real(kind=double) :: e1, e2
        real(kind=double), dimension(17) :: a
        real(kind=double) :: phi
        real(kind=double) :: y1, y2
        y1 = e1 / 250d0
        y2 = e2 / 250d0
        phi = exp (
            + a( 1) * 1d0
            + a( 2) * log(y1)
            + a( 3) * log(1d0-y1)
            + a( 4) * log(-log(y1))
            + a( 5) * log(-log(1d0-y1))
            + a( 6) * y1
            + a( 7) * log(y1)**2
            + a( 8) * log(1d0-y1)**2
            + a( 9) * log(-log(y1))**2
            + a(10) * log(-log(1d0-y1))**2
            + a(11) * y1**2
            + a(12) / log(y1)
            + a(13) / log(1d0-y1)
            + a(14) / log(-log(y1))
            + a(15) / log(-log(1d0-y1))
            + a(16) / y1
            + a(17) / (1d0-y1)
            + a( 2) * log(y2)
            + a( 3) * log(1d0-y2)
            + a( 4) * log(-log(y2))
            + a( 5) * log(-log(1d0-y2))
            + a( 6) * y2
            + a( 7) * log(y2)**2
            + a( 8) * log(1d0-y2)**2
            + a( 9) * log(-log(y2))**2
            + a(10) * log(-log(1d0-y2))**2
            + a(11) * y2**2
            + a(12) / log(y2)
            + a(13) / log(1d0-y2)
            + a(14) / log(-log(y2))
            + a(15) / log(-log(1d0-y2))
            + a(16) / y2
            + a(17) / (1d0-y2)
        )
    end function phi

```

Defines:

phi, used in chunks 110d, 112, and 116a.

```

114a <circe1_minuit1.sh 114a>≡
    #! /bin/sh
    minuit_bin='pwd'/circe1_minuit1.bin
    <Process arguments 114b>
    (
        <Define parameters 114e>
        <Fix parameters 115a>
        <Fix strategy 115b>
        <Run Minuit 115c>
    ) | eval "$minuit_bin $filter"
    <Maybe plot results 115d>
    exit 0

114b <Process arguments 114b>≡
    tmp="$IFS"
    IFS=:
    args=":$*:"
    IFS="$tmp"

114c <Process arguments 114b>+≡
    filter="| \
        awk '/STATUS=(CONVERGED|CALL LIMIT|FAILED)/ { p=1; print }; \
            /@.* \.00000 *fixed/ { next }; \
            /EDM=|CHI2|@/ && p { print }' "

114d <Process arguments 114b>+≡
    case "$args" in
        *:v:*) filter=;;
    esac

114e <Define parameters 114e>≡
    cat <<END
    set title
    CIRCE
    parameters
    1 '@ 1          ' 0.00 0.01
    2 '@ 1x         ' 0.20 0.01
    3 '@ 1(1-x)     ' 0.20 0.01
    4 '@ 11x        ' 0.00 0.01
    5 '@ 11(1-x)    ' 0.00 0.01
    6 '@ x          ' 0.00 0.01
    7 '@ 1x^2       ' 0.00 0.01
    8 '@ 1(1-x)^2   ' 0.00 0.01
    9 '@ 11x^2      ' 0.00 0.01
    10 '@ 11(1-x)^2 ' 0.00 0.01
    11 '@ x^2       ' 0.00 0.01
    12 '@ 1/1x      ' 0.00 0.01
    13 '@ 1/1(1-x)  ' 0.00 0.01
    14 '@ 1/11x     ' 0.00 0.01
    15 '@ 1/11(1-x)' 0.00 0.01

```

```

16 '@ 1/x      ' 0.00 0.01
17 '@ 1/(1-x)  ' 0.00 0.01

END

115a <Fix parameters 115a>≡
for p in 1 2 3 4 5 6 7 8 9 10 \
      11 12 13 14 15 16 17; do
case "$args" in
*: $p=*) val='echo "$args" | sed 's/.*:.'"$p"'='\([0-9.-]*\)'.*/\1/''';
        echo set parameter $p $val;
        echo fix $p;;
*: $p:*) ;;
*) echo fix $p;;
esac
done

115b <Fix strategy 115b>≡
case "$args" in
*: $0:*) echo set strategy 0;;
*: $1:*) echo set strategy 1;;
*: $2:*) echo set strategy 2;;
esac

115c <Run Minuit 115c>≡
cat <<END
migrat 10000 0.01
stop
END

115d <Maybe plot results 115d>≡
case "$args" in
*: $p:*) awk '$5 != "" { print $1, $2, $5 }' minuit.fit > chi2
        awk '$5 != "" { print $1, $5 }' minuit.fit > chix
        awk '$5 != "" { print $2, $5 }' minuit.fit > chiy
        gnuplot -geometry -0+0 plot2 >/dev/null 2>&1
esac
Uses fit 93b.

```

7.2.2 Quasi Two Dimensional

```

115e <circe1_minuit2.f90 115e>≡
! minuit2.f90 -- fitting for circe
<Copyleft notice 30a>
Uses circe 31b and minuit2 116a.

115f <circe1_minuit2.f90 115e>+≡
<Minuit2 module 116a>
<Minuit2 main program 111b>

```

```

116a <Minuit2 module 116a>≡
    module minuit2
        use kinds

        implicit none

        public :: fct
        public :: phi

        contains

        <Function to minimize 111c>
        <Function phi2 116b>
    end module minuit2

```

Defines:

minuit2, used in chunks 111b and 115e.

Uses fct 95c 111c and phi 113 116b.

```

116b <Function phi2 116b>≡
    function phi (e1, e2, a)
        real(kind=double) :: e1, e2
        real(kind=double), dimension(33) :: a
        real(kind=double) :: phi
        real(kind=double) :: y1, y2
        y1 = e1 / 250d0
        y2 = e2 / 250d0
        phi = exp (
            + a( 1) * 1d0
            + a( 2) * log(y1)
            + a( 3) * log(1d0-y1)
            + a( 4) * log(-log(y1))
            + a( 5) * log(-log(1d0-y1))
            + a( 6) * y1
            + a( 7) * log(y1)**2
            + a( 8) * log(1d0-y1)**2
            + a( 9) * log(-log(y1))**2
            + a(10) * log(-log(1d0-y1))**2
            + a(11) * y1**2
            + a(12) / log(y1)
            + a(13) / log(1d0-y1)
            + a(14) / log(-log(y1))
            + a(15) / log(-log(1d0-y1))
            + a(16) / y1
            + a(17) / (1d0-y1)
            + a(18) * log(y2)
            + a(19) * log(1d0-y2)
            + a(20) * log(-log(y2))
            + a(21) * log(-log(1d0-y2))
            + a(22) * y2
            + a(23) * log(y2)**2
        )
    end function phi

```

```

+ a(24) * log(1d0-y2)**2      &
+ a(25) * log(-log(y2))**2    &
+ a(26) * log(-log(1d0-y2))**2 &
+ a(27) * y2**2               &
+ a(28) / log(y2)             &
+ a(29) / log(1d0-y2)         &
+ a(30) / log(-log(y2))       &
+ a(31) / log(-log(1d0-y2))   &
+ a(32) / y2                  &
+ a(33) / (1d0-y2)           &
)
end function phi

```

Defines:

phi, used in chunks 110d, 112, and 116a.

```

117a <circe1_minuit2.sh 117a>≡
#! /bin/sh
minuit_bin='pwd'/circe1_minuit2.bin
<Process arguments 114b>
(
  <Define parameters (2dim) 117b>
  <Fix parameters (2dim) 118>
  <Fix strategy 115b>
  <Run Minuit 115c>
) | eval "$minuit_bin $filter"
<Maybe plot results 115d>
exit 0

```

```

117b <Define parameters (2dim) 117b>≡
cat <<END
set title
CIRCE
parameters
1  '@ 1          ' 0.00 0.01
2  '@ lx         ' 0.20 0.01
3  '@ 1(1-x)     ' 0.20 0.01
4  '@ llx        ' 0.00 0.01
5  '@ ll(1-x)    ' 0.00 0.01
6  '@ x          ' 0.00 0.01
7  '@ lx^2       ' 0.00 0.01
8  '@ 1(1-x)^2   ' 0.00 0.01
9  '@ llx^2      ' 0.00 0.01
10 '@ ll(1-x)^2  ' 0.00 0.01
11 '@ x^2        ' 0.00 0.01
12 '@ 1/lx       ' 0.00 0.01
13 '@ 1/l(1-x)   ' 0.00 0.01
14 '@ 1/llx      ' 0.00 0.01
15 '@ 1/ll(1-x)  ' 0.00 0.01
16 '@ 1/x        ' 0.00 0.01
17 '@ 1/(1-x)    ' 0.00 0.01

```

```

18 '@ ly      ' 0.20 0.01
19 '@ l(1-y)  ' 0.20 0.01
20 '@ lly     ' 0.00 0.01
21 '@ ll(1-y) ' 0.00 0.01
22 '@ y       ' 0.00 0.01
23 '@ ly^2    ' 0.00 0.01
24 '@ l(1-y)^2 ' 0.00 0.01
25 '@ lly^2   ' 0.00 0.01
26 '@ ll(1-y)^2 ' 0.00 0.01
27 '@ y^2     ' 0.00 0.01
28 '@ 1/ly    ' 0.00 0.01
29 '@ 1/l(1-y) ' 0.00 0.01
30 '@ 1/lly   ' 0.00 0.01
31 '@ 1/ll(1-y) ' 0.00 0.01
32 '@ 1/y     ' 0.00 0.01
33 '@ 1/(1-y) ' 0.00 0.01

END

118 <Fix parameters (2dim) 118>≡
for p in 1 2 3 4 5 6 7 8 9 10 \
      11 12 13 14 15 16 17 18 19 20 \
      21 22 23 24 25 26 27 28 29 30 \
      31 32 33; do
case "$args" in
*: $p=*) val='echo "$args" | sed 's/.*:.'"$p"'='\\([0-9.-]*\\):.*\\/1/'';
        echo set parameter $p $val;
        echo fix $p;;
*: $p:*) ;;
*) echo fix $p;;
esac
done

```

7.3 Version 2

8 Conclusions

I have presented a library of simple parameterizations of realistic e^\pm - and γ -beam spectra at future linear e^+e^- -colliders. The library can be used for integration and event generation. Emphasis is put on simplicity and reproducibility of the parameterizations for supporting reproducible physics simulations.

Acknowledgements

Daniel Schulte made his simulation code **Guinea-Pig** available and answered questions. Harald Anlauf and Torbjörn Sjöstrand have contributed useful suggestions. The Tesla group at DESY/Zeuthen made error estimates feasible by donating time on the multi-headed number cruncher **Hydra**. The 1996

ECFA/Desy Linear Collider Workshop got me started and provided support.
Thanks to all of them.

Identifiers

C1_ELECTRON: [11b](#), [21e](#), [31b](#), [73b](#), [80c](#), [81a](#)
 CLIC: [13a](#), [35d](#)
 C1_PHOTON: [11b](#), [31b](#), [73b](#), [80c](#), [81a](#), [87b](#)
 C1_POSITRON: [11b](#), [22](#), [81a](#)
 ILC: [13a](#), [35d](#), [69b](#), [71a](#), [71c](#), [92](#)
 JLCNLC: [13a](#), [17b](#), [18](#), [35d](#), [57b](#), [60a](#), [61a](#), [61c](#), [62c](#), [63a](#), [63c](#), [66c](#), [67c](#), [92](#)
 NACC: [13b](#), [17b](#), [34c](#), [35c](#), [35f](#), [40d](#), [41b](#), [41c](#), [41d](#), [45b](#), [45c](#), [46b](#), [46c](#), [47b](#), [47c](#),
[52a](#), [54a](#), [56c](#), [59c](#), [62a](#), [66a](#)
 NPARAM: [94a](#), [94b](#), [94d](#), [101b](#)
 SBAND: [13a](#), [35d](#), [40e](#), [41a](#), [44](#), [45a](#), [45d](#), [46a](#), [46d](#), [47a](#), [47d](#), [50a](#), [50b](#), [92](#), [107c](#),
[108c](#)
 SBNDEE: [13a](#), [34c](#), [35d](#), [41c](#), [45b](#), [46b](#), [47b](#), [49a](#), [49b](#), [92](#)
 TESLA: [13a](#), [32f](#), [33a](#), [35d](#), [40e](#), [41a](#), [44](#), [45d](#), [46d](#), [47d](#), [50a](#), [51a](#), [52b](#), [52c](#), [52d](#),
[53b](#), [54b](#), [54c](#), [55a](#), [56d](#), [56e](#), [57b](#), [57e](#), [59d](#), [60b](#), [62b](#), [62d](#), [66b](#), [67b](#), [91b](#), [92](#),
[107c](#), [108c](#), [110a](#)
 TESLEE: [13a](#), [34c](#), [35d](#), [49a](#), [92](#)
 XBAND: [13a](#), [40e](#), [41a](#), [44](#), [45a](#), [45d](#), [46a](#), [46d](#), [47a](#), [47d](#), [50a](#), [50b](#), [57e](#), [107c](#),
[108c](#)
 XBNDEE: [13a](#), [34c](#), [35d](#), [49a](#), [49b](#), [91b](#), [92](#)
 beta: [59b](#), [61e](#), [65c](#), [70b](#), [71f](#), [75a](#), [99d](#), [101a](#), [101c](#), [102d](#), [102f](#), [103e](#), [105](#), [108b](#)
 circe: [11a](#), [31b](#), [31b](#), [31b](#), [31b](#), [30b](#), [31a](#), [31b](#), [87b](#), [93b](#), [110b](#), [115e](#)
 circee: [14](#), [15c](#), [15e](#), [16a](#), [16c](#), [31b](#), [41g](#), [41h](#), [42a](#)
 circeg: [14](#), [31b](#), [42b](#), [42c](#), [43a](#)
 circel: [41f](#), [12a](#), [41e](#), [41f](#)
 circem: [33a](#), [33b](#), [33d](#), [34a](#), [34b](#), [34c](#), [36a](#), [36b](#), [36c](#), [37b](#), [37d](#), [38a](#), [38c](#), [38e](#),
[39a](#), [39b](#), [39d](#), [50c](#), [51a](#), [51b](#), [53a](#), [53b](#), [54d](#), [55a](#), [55b](#), [57a](#), [57b](#), [57e](#), [61b](#), [61c](#),
[63b](#), [63c](#), [63d](#), [65a](#), [69a](#), [69b](#), [69c](#), [79c](#), [84c](#), [86d](#), [87a](#)
 circses: [32b](#), [12b](#), [17b](#), [32b](#), [21c](#), [32a](#), [32b](#), [32b](#), [32h](#), [35f](#), [87b](#), [91b](#)
 circex: [35e](#), [35f](#)
 circgg: [14](#), [31b](#), [43b](#), [43c](#), [43d](#), [81a](#)
 d1: [15c](#), [16a](#), [15f](#), [16a](#), [41h](#), [42a](#), [42c](#), [43a](#), [43c](#), [43d](#), [74b](#), [75c](#), [75d](#), [76a](#), [76c](#),
[77b](#)
 d12: [15c](#), [15e](#), [15d](#), [15e](#)
 d2: [15c](#), [16c](#), [16b](#), [16c](#), [41h](#), [42a](#), [42c](#), [43a](#), [43c](#), [43d](#), [74b](#), [75c](#), [75d](#), [76a](#), [76c](#),
[77b](#)
 d12a: [16e](#), [16g](#), [17a](#)
 fct: [93b](#), [95b](#), [95c](#), [110d](#), [111a](#), [111b](#), [111c](#), [116a](#)
 fit: [93b](#), [94d](#), [111a](#), [111b](#), [115d](#)
 fixerr: [97b](#), [97c](#), [97d](#)
 gauss1: [89f](#), [15c](#), [89f](#), [89e](#), [89f](#), [89f](#)
 gauss2: [90d](#), [15c](#), [16e](#), [90d](#), [90c](#), [90d](#)
 gaussx: [90b](#), [90a](#), [90b](#), [90b](#), [90d](#)
 gethst: [95e](#), [96a](#), [96b](#)
 girce: [80c](#), [20a](#), [80c](#), [80b](#), [80c](#)
 girceb: [84b](#), [82a](#), [82d](#), [83c](#), [84a](#), [84b](#), [84c](#), [86b](#)

gircee: 20c, 20d, 81e, 21d, 80c, 81d, 81e
 girceg: 20c, 80c, 82b, 82c
 gircgg: 20c, 80c, 83a, 83b, 83c
 kirke: 73a, 73b
 kirkee: 17a, 73b, 74a, 74b, 75c
 kirkeg: 73b, 75d, 76b, 76c
 kirkgg: 73b, 76a, 77a, 77b
 minuit1: 110d, 111a
 minuit2: 111b, 115e, 116a
 phi: 110d, 112c, 112d, 113, 116a, 116b
 phie: 99b, 100b, 100d, 100e, 103b
 phig: 99b, 100b, 100f, 100g, 103b
 pslice: 103b, 103d, 103e
 random: 20d, 21a, 21b, 21d
 rng_call: 79b, 79c
 rng_generate: 79f, 80a
 rng_proc: 79a, 79d, 79d
 rng_type: 79a, 79e, 79f, 80a
 scale: 98c, 98d, 98e
 sigma: 15a, 15b, 15c, 15e, 16a, 16c, 16e, 17a, 18, 20d
 sumsqu: 97e, 98a, 98b

Refinements

⟨API documentation 11a⟩
 ⟨Abstract interfaces 79d⟩
 ⟨Abstract types 79f⟩
 ⟨Accelerator codes 13a⟩
 ⟨Alternative: Local variables for circes 75b⟩
 ⟨Alternative: Subroutines 77c⟩
 ⟨Alternative: Update circe1 parameters 75a⟩
 ⟨Calculate f 112c⟩
 ⟨Calculate f ($v1$) 99b⟩
 ⟨Calculate ∇f 99a⟩
 ⟨Calculate version 1 of the e^+e^- distribution 42a⟩
 ⟨Calculate version 1 of the $e^\pm\gamma$ distribution 43a⟩
 ⟨Calculate version 1 of the $\gamma\gamma$ distribution 43d⟩
 ⟨Calculate version 1 of the non-singular e^+e^- distribution 75c⟩
 ⟨Calculate version 1 of the non-singular $e^\pm\gamma$ distribution 75d⟩
 ⟨Calculate version 1 of the non-singular $\gamma\gamma$ distribution 76a⟩
 ⟨Check **a** and **b** 84c⟩
 ⟨Code that has to be at the top 126c⟩
 ⟨Copleft notice 30a⟩
 ⟨Declaration: circe1 parameters 32d⟩
 ⟨Declaration of **accnam** 35c⟩
 ⟨Declare **NPARAM** 94a⟩
 ⟨Declare arguments 94c⟩
 ⟨Declare parameters 94b⟩
 ⟨Define parameters 114e⟩

<Define parameters (2dim) 117b>
 <EPS & PWR 16d>
 <Event generation 21d>
 <Fix parameters 115a>
 <Fix parameters (2dim) 118>
 <Fix strategy 115b>
 <Fixup errors 97b>
 <Function call stub 89a>
 <Function phi1 113>
 <Function phi2 116b>
 <Function to minimize 111c>
 <Gauss integration 15c>
 <Gaussian weights 91a>
 <Generate a trial \mathbf{x} and calculate its weight \mathbf{w} 85b>
 <Generate version 1 of the e^+e^- distribution 82a>
 <Generate version 1 of the $e^\pm\gamma$ distribution 82d>
 <Generate version 1 of the $\gamma\gamma$ distribution 83c>
 <Give up on f 100a>
 <Initialization check 32h>
 <Initialization of **accnam** 35d>
 <Initializations for **circes** 35b>
 <Initialize **circe1** parameters 33a>
 <Initialize event generator 21c>
 <Initialize parameters for **circe1_fit.f90** 95a>
 <Linearly interpolate energies 57e>
 <Linearly interpolate energies for ILC 2013 69c>
 <Linearly interpolate energies for JLC/NLC 2002 63d>
 <Linearly interpolate energies for NLC H 2002 65a>
 <Iterate programming example 126a>
 <Load parameters 94d>
 <Local variables for **circes** 33c>
 <Local variables for **fct** 112b>
 <Local variables for **fct** ($v1$) 95f>
 <Log energy mapping 39d>
 <Log revision mapping 38a>
 <Main module 30c>
 <Map roots to **e** 38c>
 <Map roots to **e** at low energies 55b>
 <Maybe plot results 115d>
 <Minuit1 main program 111a>
 <Minuit2 main program 111b>
 <Minuit1 module 110d>
 <Minuit2 module 116a>
 <Module subroutines 31b>
 <Monte Carlo integration 20d>
 <Normalize 97e>
 <Other code 126b>
 <Other variables in **sample** 19>
 <Part one of Gaussian integration 88>
 <Part three of Gaussian integration 89d>

<Part two of Gaussian integration 89b>
 <Particle codes 11b>
 <Private parameters 37c>
 <Process arguments 114b>
 <Public subroutines 31a>
 <Public types 79e>
 <RNG dummy arguments 78>
 <RNG dummy declarations 79a>
 <RNG: generate **u** 79b>
 <Read continuum, summing in **s** 96c>
 <Read data from file 95e>
 <Read double δ , summing in **t** 97a>
 <Read input data 112a>
 <Read input data (**v1**) 95d>
 <Read single δ , summing in **t** 96d>
 <Run Minuit 115c>
 <Sample output 18>
 <Second Gauss integration 16e>
 <Select particles **p1** and **p2** 81a>
 <Set up best value for **t** 86c>
 <Set up **girceb** parameters 85a>
 <Update **circe1** parameters 33b>
 <Update version 3 and 4 derived parameters in **circe1** parameters 51a>
 <Update version 1 derived parameters in **circe1** parameters 37d>
 <Update version 10 derived parameters in **circe1** parameters 69a>
 <Update version 5 derived parameters in **circe1** parameters 53a>
 <Update version 6 derived parameters in **circe1** parameters 54d>
 <Update version 7 derived parameters in **circe1** parameters 57a>
 <Update version 8 derived parameters in **circe1** parameters 61b>
 <Update version 9 derived parameters in **circe1** parameters 63b>
 <Version 2 has been retired 50c>
 <Warn that no parameter set has been endorsed for e^-e^- yet 36a>
 <Write output 112d>
 <Write output (**v1**) 101a>
 <4-byte aligned part of **circe1** parameters 32f>
 <8-byte aligned part of **circe1** parameters 32e>
 <**circe1.f90** 29>
 <**circe1_fit.f90** 93b>
 <**circe1_fit.f90**: public 95b>
 <**circe1_fit.f90**: subroutines 95c>
 <**circe1_fit.sh** 106>
 <**circe.h** 30b>
 <**circe1_minuit1.f90** 110b>
 <**circe1_minuit2.f90** 115e>
 <**circe1_minuit1.sh** 114a>
 <**circe1_minuit2.sh** 117a>
 <**circe1_plot.f90** 87b>
 <**circe1_sample.f90** 17b>
 <**circe1_sample.f90**: public 15a>
 <**circe1_sample.f90**: subroutines 15b>

<else handle invalid versions 37b>
<formats for circes 38d>
<params.f90 91b>
<uniform deviate on [0, 1] (never defined)>
<x1m, x2m kludge, part 1 81b>
<x1m, x2m kludge, part 2 81c>

Index

inefficiencies, [18](#), [80](#)

System dependencies, [102](#)

References

- [1] H. Murayama and M. E. Peskin, SLAC-PUB-7149, to appear in *Ann. Rev. Nucl. Part. Sci.*; P. Zerwas, DESY 94-001-REV.
- [2] P. Chen and R. J. Noble, SLAC-PUB-4050; M. Bell and J. S. Bell, Part. Accl. **24**, 1 (1988); R. Blankenbecler and S. D. Drell, Phys. Rev. Lett. **61**, 2324 (1988); P. Chen and K. Yokoya, Phys. Rev. Lett. **61**, 1101 (1988); M. Jacob and T. T. Wu, Nucl. Phys. **B303**, 389 (1988); V. N. Baier, V. M. Katkov, and V. M. Strakovenkov, Nucl. Phys. **B328**, 387 (1989); R. Blankenbecler, S. D. Drell, and N. Kroll, Phys. Rev. **D40**, 2462 (1989); P. Chen and V. L. Telnov, Phys. Rev. Lett. **63**, 1796 (1989).
- [3] K. Yokoya, KEK 85-9, KEK.
- [4] P. Chen *et al.*, Nucl. Inst. Meth. **A355**, 107 (1995).
- [5] D. Schulte, Ph.D. thesis, in preparation.
- [6] R. B. Palmer, Ann. Rev. Nucl. Part. Sci. **40**, 529 (1990).
- [7] P. Chen, Phys. Rev. **D46**, 1186 (1992).
- [8] Tesla Collaboration, Conceptual Design Report, in preparation.
- [9] Desy-Darmstadt Linear Collider Collaboration, Conceptual Design Report, in preparation.
- [10] JLC Group, KEK Report 92-16.
- [11] NLC ZDR Design Group, SLAC-Report-474.
- [12] NLC ZDR Design Group and NLC Physics Working Groups, SLAC-Report-485.
- [13] Particle Data Group, Phys. Rev. **D50**, 1173 (1994).
- [14] G. Altarelli, R. Kleiss, and C. Verzegnassi, CERN Yellow Report 89-08.
- [15] F. James and M. Roos, *MINUIT, Function Minimization and Error Analysis, Release 89.12j*, CERN, Geneva, 1989.
- [16] H. Anlauf, IKDA 96/6.
- [17] H. Anlauf, private communication.
- [18] A. Atkinson and J. Whittaker, Appl. Stat. **28**, 90 (1979).
- [19] D. E. Knuth, *Literate Programming*, Vol. 27 of *CSLI Lecture Notes* (Center for the Study of Language and Information, Leland Stanford Junior University, Stanford, CA, 1991).
- [20] D. E. Knuth, *T_EX: The Program*, Vol. B of *Computers & Typesetting* (Addison-Wesley, Reading, Mass., 1986).
- [21] D. E. Knuth, *METAFONT: The Program*, Vol. D of *Computers & Typesetting* (Addison-Wesley, Reading, Mass., 1986).
- [22] N. Ramsey, IEEE Software **11**, 97 (1994).

A Literate Programming

A.1 Paradigm

I have presented the sample code in this paper using the *literate programming* paradigm. This paradigm has been introduced by Donald Knuth [19] and his programs T_EX [20] and METAFONT [21] provide excellent examples of the virtues of literate programming. Knuth summarized his intention as follows ([19], p. 99)

“Let us change our traditional attitude to the construction of programs. Instead of imagining that our main task is to instruct a *computer* what to do, let us concentrate rather on explaining to *human beings* what we want a computer to do.”

Usually, literate programming uses two utility programs to produce two kinds of files from the source

tangle produces the computer program that is acceptable to an “illiterate” (Fortran, C, etc.) compiler. This process consists of stripping documentation and reordering code. Therefore it frees the author from having to present the code in the particular order enforced by a compiler for purely technical reasons. Instead, the author can present the code in the order that is most comprehensible.

weave produces a documents that describes the program. Extensive cross referencing of the code sections is usually provided, which has been suppressed in this paper. If a powerful typesetting system (such a T_EX) is used, the document can present the algorithms in clear mathematical notation alongside the code. These features improve readability and maintainability of scientific code immensely.

A.2 Practice

Circe1 uses the **noweb** [22] system. This system has the advantage to work with any traditional programming language and support the essential features described in section A.1 with minimal effort. **noweb**’s **tangle** program only reorders the code sections, but does not reformat them. Therefore its output can be used just like any other “illiterate” program.

The examples above should be almost self-explaining, but in order to avoid any ambiguities, I give another example:

```
126a <Literate programming example 126a>≡  
      <Code that has to be at the top 126c>  
      <Other code 126b>
```

I can start the presentation with the first line of the “other code”:

```
126b <Other code 126b>≡  
      line 1 of the other code
```

If appropriate, the first line of the code that has to appear *before* the other code can be presented later:

```
126c <Code that has to be at the top 126c>≡  
      line 1 of the code at the top
```

Now I can augment the sections:

```
127a <Other code 126b>+≡  
    line 2 of the other code  
127b <Code that has to be at the top 126c>+≡  
    line 2 of the code at the top
```

The complete “program” will be presented to the compiler as

```
line 1 of the code at the top  
line 2 of the code at the top  
line 1 of the other code  
line 2 of the other code
```

The examples in section 3.1.1 show that this reordering is particularly useful for declaring variables when they are first used (rather than at the beginning) and for zooming in on code inside of loops.

B Fortran Name Space

In addition to the ten procedures and one `common` block discussed in section 3

- `circe`, `circee`, `circeg`, `circgg`,
- `girce`, `gircee`, `girceg`, `gircgg`,
- `circes`, `circel`, `/circom/`,

there are two more globally visible functions which are used internally:

- `circem`: error message handler,
- `girceb`: efficient Beta distribution generator.

Even if the `/circom/` is globally visible, application programs *must not* manipulate it directly. The `circes` subroutine is provided for this purpose and updates some internal parameters as well.

With features from the current Fortran standard (Fortran90), I could have kept the last two functions and the `common` block private.

Application programs wishing to remain compatible with future versions of `Circe1` must not use `common` blocks or procedures starting with `circe` or `girce`.

C Updates

Information about updates can be obtained

- on the World Wide Web:

`http:http://projects.hepforge.org/whizard/`

Contributions of results from other simulation programs and updated accelerator designs are welcome at

`ohl@physik.uni-wuerzburg.de`