# Package ‘cmvnorm’

October 12, 2022

**Type** Package

**Title** The Complex Multivariate Gaussian Distribution

**Version** 1.0-7

**Depends** emulator (>= 1.2-21)

**Suggests** knitr

**Imports** elliptic

**Maintainer** Robin K. S. Hankin <hankin.robin@gmail.com>

**Description** Various utilities for the complex multivariate Gaussian distribution and complex Gaussian processes.

**VignetteBuilder** knitr



**URL** https://github.com/RobinHankin/cmvnorm

**BugReports** https://github.com/RobinHankin/cmvnorm/issues

**NeedsCompilation** no

**Author** Robin K. S. Hankin [aut, cre] (<https://orcid.org/0000-0001-5982-0415>)

**Repository** CRAN

**Date/Publication** 2022-01-31 00:00:02 UTC

## R topics documented:

cmvnorm-package *The Complex Multivariate Gaussian Distribution*

## Description

Various utilities for the complex multivariate Gaussian distribution and complex Gaussian processes.

## Details

The DESCRIPTION file:

| | |
|---|---|
| Package: | cmvnorm |
| Type: | Package |
| Title: | The Complex Multivariate Gaussian Distribution |
| Version: | 1.0-7 |
| Authors@R: | person(given=c("Robin", "K. S."), family="Hankin", role = c("aut","cre"), email="hankin.robin@gmail.co |
| Depends: | emulator (>= 1.2-21) |
| Suggests: | knitr |
| Imports: | elliptic |
| Maintainer: | Robin K. S. Hankin <hankin.robin@gmail.com> |
| Description: | Various utilities for the complex multivariate Gaussian distribution and complex Gaussian processes. |
| VignetteBuilder: | knitr |
| License: | GPL-2 |
| URL: | https://github.com/RobinHankin/cmvnorm |
| BugReports: | https://github.com/RobinHankin/cmvnorm/issues |
| Author: | Robin K. S. Hankin [aut, cre] (<https://orcid.org/0000-0001-5982-0415>) |

Index of help topics:

```
Im<-                    Manipulate real or imaginary components of an
                        object
Mvcnorm                 Multivariate complex Gaussian density and
                        random deviates
cmvnorm-package         The Complex Multivariate Gaussian Distribution
corr_complex            Complex Gaussian processes
isHermitian             Is a Matrix Hermitian?
var                     Variance and standard deviation of complex
                        vectors
wishart                 The complex Wishart distribution
```

Generalizing the real multivariate Gaussian distribution to the complex case is not straightforward but one common approach is to replace the real symmetric variance matrix with a Hermitian positive-definite matrix. The **cmvnorm** package provides some functionality for the resulting density function.

### Author(s)

NA

Maintainer: Robin K. S. Hankin <hankin.robin@gmail.com>

### References


- N. R. Goodman 1963. "Statistical analysis based on a certain multivariate complex Gaussian distribution". *The Annals of Mathematical Statistics*. 34(1): 152–177
- R. K. S. Hankin 2015. "The complex multivariate Gaussian distribution". *R News*, volume 7, number 1.


### Examples

```
S1 <- 4+diag(5)
S2 <- S1
S2[1,5] <- 4+1i
S2[5,1] <- 4-1i   # Hermitian


rcmvnorm(10,sigma=S1)
rcmvnorm(10,mean=rep(1i,5),sigma=S2)

dcmvnorm(rep(1,5),sigma=S2)
```

---

## corr_complex *Complex Gaussian processes*

---

### Description

Various utilities for investigating complex Gaussian processes

### Usage

```
corr_complex(z1, z2 = NULL, distance.function = complex_CF, means =
NULL, scales = NULL, pos.def.matrix = NULL)
complex_CF(z1,z2, means, pos.def.matrix)
scales.likelihood.complex(pos.def.matrix, scales, means,  zold, z,
               give_log = TRUE, func = regressor.basis)
interpolant.quick.complex(x, d, zold, Ainv, scales = NULL, pos.def.matrix = NULL,
               means=NULL, func = regressor.basis, give.Z = FALSE,
               distance.function = corr_complex, ...)
```

### Arguments

`z, z1, z2` Points in $C^n$

`distance.function`
Function giving the (complex) covariance between two points in $C^n$

`means, pos.def.matrix, scales`
In function `complex_CF()`, the mean and covariance matrix of the distribution whose characteristic function is used to give the covariance matrix; `scales` is used to specify the diagonal of `pos.def.matrix` if the off-diagonal elements are zero

`zold, d, give_log, func, x, Ainv, give.Z,...`
Direct analogues of the arguments in `interpolant()` and `scales.likelihood()` in the **emulator** package

### Details

- Function `complex_CF()` returns a (slightly reparameterized) characteristic function of a complex Gaussian distribution. The covariance is given by

$$c(\mathbf{t}) = \exp(i\mathrm{Re}(\mathbf{t}^*\mu) - \mathbf{t}^*B\mathbf{t})$$

  where $\mathbf{t} = \mathbf{x} - \mathbf{x}'$ is interpreted as the distance between two observations, $\mu$ is the mean of the distribution (which is in general a complex vector), and $B$ a positive-definite matrix.
- Function `corr_complex()` is the complex analogue of `corr.matrix()`. It returns a matrix with entry $(i, j)$ equal to the covariance of the process at observation $i$ and observation $j$, or $\mathrm{cov}(\eta(\mathbf{x}_i), \eta(\mathbf{x}_j))$. The elements are calculated by `complex_CF()`.
  This function includes only a single method, that of nested calls to `apply()`. I could not figure out how to generalize method 1 of `corr.matrix()` to the complex case.
- Function `scales.likelihood.complex()` is a complex version of `scales.likelihood()` which takes a positive definite matrix and a mean. The formula used is

$$(\sigma^2)^{-(n-q)}|A|^{-1}|H^*A^{-1}H|^{-1}$$

  . Here and elsewhere, $A^*$ means the complex conjugate of the transpose.
- Function `interpolant.quick.complex()` is a complex version of `interpolant.quick()`.

$$\mathbf{h}(\mathbf{x})^*\hat{\beta} + \mathbf{t}(\mathbf{x})^*A^{-1}(\mathbf{y} - H\hat{\beta})$$

  This is the complex version of Oakley's equation 2.30 or Hankin's equation 5.

More details are given in the package vignette.

### Author(s)

Robin K. S. Hankin

### References

- Hankin, R. K. S. 2005. "Introducing BACCO, an R bundle for Bayesian Analysis of Computer Code Output", *Journal of Statistical Software*, 14(15)
- J. Oakley 1999. *Bayesian uncertainty analysis for complex computer codes*, PhD thesis, University of Sheffield.

## Examples

```
complex_CF(c(1,1i),c(1,-1i),means=c(1i,1i),pos.def.matrix=diag(2))

V <- latin.hypercube(7,2,complex=TRUE)

cm <- c(1,1+1i)                     # "complex mean"
cs <- matrix(c(2,1i,-1i,1),2,2)   # "complex scales"
tb <- c(1,1i,1-1i)                     # "true beta"

A <- corr_complex(V,means=cm,pos.def.matrix=cs)
Ainv <- solve(A)
z <- drop(rcmvnorm(n=1,mean=regressor.multi(V) %*% tb, sigma=A))


betahat.fun(V,Ainv,z)    # should be close to 'tb'

#scales.likelihood.complex(cs,cm,V,z)   # log-likelihood evaluated true parameters



interpolant.quick.complex(x=0.1i+V[1:3,],d=z,zold=V,Ainv=Ainv,pos.def.matrix=cs,means=cm)
```

---

`isHermitian` *Is a Matrix Hermitian?*

---

## Description

Returns `TRUE` if a matrix is Hermitian or Hermitian positive-definite

## Usage

```
isHermitian(x, tol = 100 * .Machine$double.eps)
ishpd(x,tol= 100 * .Machine$double.eps)
zapim(x,tol= 100 * .Machine$double.eps)
```

## Arguments

| | |
|---|---|
| `x` | A square matrix |
| `tol` | Tolerance for numerical scruff |

## Details

Functions `isHermitian()` and `ishpd()` return a Boolean, indicating whether the argument is Hermitian or Hermitian positive definite respectively. Function `zapim()` zaps small imaginary parts of a vector, returning real if all elements are so zapped.

## Author(s)

Robin K. S. Hankin

## Examples

```
v <- 2^(1:30)
zapim(v+1i*exp(-v))


ishpd(matrix(c(1,0.1i,-0.1i,1),2,2))   # should be TRUE
isHermitian(matrix(c(1,3i,-3i,1),2,2)) # should be TRUE
ishpd(rcwis(6,2))                      # should be TRUE
```

---

Mvcnorm *Multivariate complex Gaussian density and random deviates*

---

## Description

Density function and a random number generator for the multivariate complex Gaussian distribution.

## Usage

```
rcnorm(n)
dcmvnorm(z, mean, sigma, log = FALSE)
rcmvnorm(n, mean = rep(0, nrow(sigma)), sigma = diag(length(mean)),
    method = c("svd", "eigen", "chol"),
    tol= 100 * .Machine$double.eps)
```

## Arguments

| | |
|---|---|
| z | Complex vector or matrix of quantiles. If a matrix, each row is taken to be a quantile |
| n | Number of observations |
| mean | Mean vector |
| sigma | Covariance matrix, Hermitian positive-definite |
| tol | numerical tolerance term for verifying positive definiteness |
| log | In dcmvnorm(), Boolean with default FALSE meaning to return the Gaussian density function, and TRUE meaning to return the logarithm |
| method | Specifies the decomposition used to determine the positive-definite matrix square root of sigma. Possible methods are eigenvalue decomposition ("eigen", default), and singular value decomposition ("svd") |

### Details

Function dcmvnorm() is the density function of the complex multivariate normal (Gaussian) distribution:

$$p(\mathbf{z}) = \frac{\exp\left(-\mathbf{z}^{*}\Gamma\mathbf{z}\right)}{|\pi\Gamma|}$$

Function rcnorm() is a low-level function designed to generate observations drawn from a standard complex Gaussian. Function rcmvnorm() is a user-friendly wrapper for this.

### Author(s)

Robin K. S. Hankin

### References

N. R. Goodman 1963. "Statistical analysis based on a certain multivariate complex Gaussian distribution". *The Annals of Mathematical Statistics*. 34(1): 152–177

### Examples

```
S <- emulator::cprod(rcmvnorm(3,mean=c(1,1i),sigma=diag(2)))

rcmvnorm(10,sigma=S)
rcmvnorm(10,mean=c(0,1+10i),sigma=S)


# Now try and estimate the mean (viz 1,1i) and variance (S) from a
#  random sample:


n <- 101
z <- rcmvnorm(n,mean=c(0,1+10i),sigma=S)
xbar <- colMeans(z)
Sbar <- cprod(sweep(z,2,xbar))/n
```

---

setreal *Manipulate real or imaginary components of an object*

---

### Description

Manipulate real or imaginary components of an object

### Usage

```
Im(x) <- value
Re(x) <- value
```

### Arguments

| | |
|---|---|
| `x` | Complex-valued object |
| `value` | Real-valued object |

### Author(s)

Robin K. S. Hankin

### Examples

```
A <- matrix(c(1,0.1i,-0.1i,1),2,2)
Im(A) <- Im(A)*3
Re(A) <- matrix(c(5,2,2,5),2,2)
```

---

`var` *Variance and standard deviation of complex vectors*

---

### Description

Complex generalizations of `stats::sd()` and `stats::var()`

### Usage

```
var(x, y=NULL, na.rm=FALSE,use)
sd(x, na.rm=FALSE)
```

### Arguments

| | |
|---|---|
| `x,y` | Complex vector or matrix |
| `na.rm` | Boolean with default `FALSE` meaning to leave `NA` values present and `TRUE` meaning to remove them |
| `use` | Ignored |

## Details

Intended to be broadly compatible with `stats::sd()` and `stats::var()`.

If given real values, `var()` and `sd()` return the variance and standard deviation as per ordinary real analysis. If given complex values, returns the complex generalization in which Hermitian transposes are used.

If `z` is a complex matrix, `var(z)` returns the variance of the rows.

These functions use $n - 1$ on the denominator purely for consistency with `stats::var()` (for the record, I disagree with the rationale for $n - 1$).

## Author(s)

Robin K. S. Hankin

## Examples

```
sd(rcnorm(10)) # imaginary component suppressed by zapim()

var(rcmvnorm(1e5,mean=c(0,0)))
```

---

`wishart` *The complex Wishart distribution*

---

## Description

Returns an observation drawn from the complex Wishart distribution. To sample from the inverse complex Wishart distribution (or indeed the complex inverse Wishart distribution), use `solve(rcwis(...))`.

## Usage

```
rcwis(n, S)
```

## Arguments

| | |
|---|---|
| `n` | Integer; degrees of freedom |
| `S` | Variance matrix. If an integer, use `diag(nrow=S)` |

## Value

Returns a (semi-) positive definite Hermitian matrix the same size as argument `S`

## Note

The first argument of `rcwis()` is `n`, by universal statistics convention. But in the R world, functions returning random observations (such as `runif()`) generally reserve argument `n` for the number of observations to return. Although `rchisq()` uses `df` for the number of degrees of freedom.

## Author(s)

Robin K. S. Hankin

## Examples

```
rcwis(10,2)
eigen(rcwis(7,3),TRUE,TRUE)   # all positive
eigen(rcwis(3,7),TRUE,TRUE)   # 4 positive, 3 zero

rcwis(10,rcwis(10,3))
```

# Index