# Package 'GWmodel'

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Type Package

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Title Geographically-Weighted Models

**Depends** R (>= 3.0.0), robustbase, sp (> 1.4-0), Rcpp (>= 1.0.12)

Imports methods, sf, grDevices, spacetime, spdep, spatialreg, FNN

LinkingTo Rcpp, RcppArmadillo, RcppEigen

Suggests mvoutlier, RColorBrewer, gstat, spData

Description Techniques from a particular branch of spatial statistics, termed geographically-weighted (GW) models. GW models suit situations when data are not described well by some global model, but where there are spatial regions where a suitably localised calibration provides a better description. 'GWmodel' includes functions to calibrate: GW summary statistics (Brunsdon et al., 2002)<doi:10.1016/s0198-9715(01)00009-6>, GW principal components analysis (Harris et al., 2011)<doi:10.1080/13658816.2011.554838>, GW discriminant analysis (Brunsdon et al., 2007)<doi:10.1111/j.1538-4632.2007.00709.x> and various forms of GW regression (Brunsdon et al., 1996)<doi:10.1111/j.1538-4632.1996.tb00936.x>; some of which are provided in basic and robust (outlier resistant) forms.

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License GPL (>= 2)

**Repository** CRAN

URL http://gwr.nuim.ie/

NeedsCompilation yes

SystemRequirements GNU make

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# Contents

GWmodel-package	3
bw.ggwr	4
bw.gtwr	6
bw.gwda	7
bw.gwpca	9
bw.gwr	0
bw.gwr.lcr	1
bw.gwss.average	3
DubVoter	4
EWHP	5
EWOutline	6
Georgia	6
GeorgiaCounties	7
ggwr.basic	8
ggwr.cv	0
ggwr.cv.contrib	1
gtwr	2
gw.dist	4
gw.pcplot	6
gw.weight	7
gwda	8
gwpca	0
gwpca.check.components	3
gwpca.cv	4
gwpca.cv.contrib	5
gwpca.glyph.plot	6
gwpca.montecarlo.1	6
gwpca.montecarlo.2	8
gwr.basic	0
gwr.bootstrap	4
gwr.collin.diagno	8
gwr.cv	9
gwr.cv.contrib	0
gwr.hetero	2
gwr.lcr	3
gwr.lcr.cv	6
gwr.lcr.cv.contrib	7
gwr.mink.approach	8
gwr.mink.matrixview	0
gwr.mink.pval	1
gwr.mixed	2

gwr.model.selection	64
gwr.model.sort	66
gwr.model.view	67
gwr.montecarlo	68
gwr.multiscale	69
gwr.predict	73
gwr.robust	75
gwr.scalable	78
gwr.t.adjust	79
gwr.write	80
gwss	81
gwss.montecarlo	83
LondonBorough	85
LondonHP	85
st.dist	86
USelect	88
	89

# Index

GWmodel-package Geographically-Weighted Models

#### Description

In GWmodel, we introduce techniques from a particular branch of spatial statistics, termed geographicallyweighted (GW) models. GW models suit situations when data are not described well by some global model, but where there are spatial regions where a suitably localised calibration provides a better description. GWmodel includes functions to calibrate: GW summary statistics, GW principal components analysis, GW discriminant analysis and various forms of GW regression; some of which are provided in basic and robust (outlier resistant) forms. In particular, the high-performence computing technologies, including multi-thread and CUDA techniques are started to be adopted for efficient calibrations.

# Details

Package:	GWmodel
Type:	Package
Version:	2.4-1
Date:	2024-09-06
License:	GPL (>=2)
LazyLoad:	yes

3

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Beta versions can always be found at https://github.com/lbb220/GWmodel, which includes all the newly developed functions for GW models.

For latest tutorials on using GWmodel please go to: https://rpubs.com/gwmodel

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#### References

Gollini I, Lu B, Charlton M, Brunsdon C, Harris P (2015) GWmodel: an R Package for exploring Spatial Heterogeneity using Geographically Weighted Models. Journal of Statistical Software, 63(17):1-50, doi: 10.18637/jss.v063.i17

Lu B, Harris P, Charlton M, Brunsdon C (2014) The GWmodel R Package: further topics for exploring Spatial Heterogeneity using Geographically Weighted Models. Geo-spatial Information Science 17(2): 85-101, doi: 10.1080/10095020.2014.917453

Lu, B., Hu, Y., Yang, D., Liu, Y., Ou, G., Harris, P., Brunsdon, C., Comber, A., Dong, G., 2024. Gwmodels: A standalone software to train geographically weighted models. Geo-spatial Information Science, 1-23.

Lu, B., Hu, Y., Murakami, D., Brunsdon, C., Comber, A., Charlton, M., Harris, P., 2022. Highperformance solutions of geographically weighted regression in r. Geo-spatial Information Science 25 (4), 536-549.

bw.ggwr

Bandwidth selection for generalised geographically weighted regression (GWR)

# Description

A function for automatic bandwidth selection to calibrate a generalised GWR model

#### Usage

```
bw.ggwr(formula, data, family ="poisson", approach="CV",
kernel="bisquare",adaptive=FALSE, p=2, theta=0, longlat=F,dMat)
```

#### Note

# bw.ggwr

#### Arguments

formula	Regression model formula of a formula object
data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>
family	a description of the error distribution and link function to be used in the model, which can be specified by "poisson" or "binomial"
approach	specified by CV for cross-validation approach or by AIC corrected (AICc) approach
kernel	function chosen as follows:
	gaussian: wgt = $exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
	tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
adaptive	if TRUE calculate an adaptive kernel where the bandwidth corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist

# Value

Returns the adaptive or fixed distance bandwidth

### Note

For a discontinuous kernel function, a bandwidth can be specified either as a fixed (constant) distance or as a fixed (constant) number of local data (i.e. an adaptive distance). For a continuous kernel function, a bandwidth can be specified either as a fixed distance or as a 'fixed quantity that reflects local sample size' (i.e. still an 'adaptive' distance but the actual local sample size will be the sample size as functions are continuous). In practise a fixed bandwidth suits fairly regular sample configurations whilst an adaptive bandwidth suits highly irregular sample configurations. Adaptive bandwidths ensure sufficient (and constant) local information for each local calibration. This note is applicable to all GW models

# Author(s)

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bw.gtwr

# Description

A function for automatic bandwidth selection to calibrate a GTWR model

# Usage

# Arguments

formula	Regression model formula of a formula object
data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>
obs.tv	a vector of time tags for each observation, which could be numeric or of POSIXIt class
approach	specified by CV for cross-validation approach or by AIC corrected (AICc) approach
kernel	function chosen as follows:
	gaussian: wgt = $exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = $(1-(vdist/bw)^2)^2$ if vdist < bw, wgt=0 otherwise;
	tricube: $wgt = (1-(vdist/bw)^3)^3$ if $vdist < bw$ , $wgt=0$ otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
adaptive p	the number of nearest neighbours (i.e. adaptive distance); default is FALSE,
·	the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
p	the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance) the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
p theta	the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance) the power of the Minkowski distance, default is 2, i.e. the Euclidean distance an angle in radians to rotate the coordinate system, default is 0
p theta longlat	the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance) the power of the Minkowski distance, default is 2, i.e. the Euclidean distance an angle in radians to rotate the coordinate system, default is 0 if TRUE, great circle distances will be calculated
p theta longlat lamda	the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance) the power of the Minkowski distance, default is 2, i.e. the Euclidean distance an angle in radians to rotate the coordinate system, default is 0 if TRUE, great circle distances will be calculated an parameter between 0 and 1 for calculating spatio-temporal distance
p theta longlat lamda t.units	the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance) the power of the Minkowski distance, default is 2, i.e. the Euclidean distance an angle in radians to rotate the coordinate system, default is 0 if TRUE, great circle distances will be calculated an parameter between 0 and 1 for calculating spatio-temporal distance character string to define time unit an parameter between 0 and PI for calculating spatio-temporal distance, see de-
p theta longlat lamda t.units ksi	the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance) the power of the Minkowski distance, default is 2, i.e. the Euclidean distance an angle in radians to rotate the coordinate system, default is 0 if TRUE, great circle distances will be calculated an parameter between 0 and 1 for calculating spatio-temporal distance character string to define time unit an parameter between 0 and PI for calculating spatio-temporal distance, see de- tails in Wu et al. (2014)

# bw.gwda

# Value

Returns the adaptive or fixed distance bandwidth

# Note

The function is developed according to the articles by Huang et al. (2010) and Wu et al. (2014).

# Author(s)

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#### References

Huang, B., Wu, B., & Barry, M. (2010). Geographically and temporally weighted regression for modeling spatio-temporal variation in house prices. International Journal of Geographical Information Science, 24, 383-401.

Wu, B., Li, R., & Huang, B. (2014). A geographically and temporally weighted autoregressive model with application to housing prices. International Journal of Geographical Information Science, 28, 1186-1204.

Fotheringham, A. S., Crespo, R., & Yao, J. (2015). Geographical and Temporal Weighted Regression (GTWR). Geographical Analysis, 47, 431-452.

bw.gwda

Bandwidth selection for GW Discriminant Analysis

#### Description

A function for automatic bandwidth selection for GW Discriminant Analysis using a cross-validation approach only

#### Usage

### Arguments

formula	Model formula of a formula object
data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>
COV.gw	if true, localised variance-covariance matrix is used for GW discriminant analy- sis; otherwise, global variance-covariance matrix is used
mean.gw	if true, localised mean is used for GW discriminant analysis; otherwise, global mean is used

prior.gw	if true, localised prior probability is used for GW discriminant analysis; other- wise, fixed prior probability is used
prior	a vector of given prior probability
wqda	if TRUE, a weighted quadratic discriminant analysis will be applied; otherwise a weighted linear discriminant analysis will be applied
kernel	function chosen as follows:
	gaussian: wgt = $exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
	tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist

# Value

Returns the adaptive or fixed distance bandwidth.

#### Note

For a discontinuous kernel function, a bandwidth can be specified either as a fixed (constant) distance or as a fixed (constant) number of local data (i.e. an adaptive distance). For a continuous kernel function, a bandwidth can be specified either as a fixed distance or as a 'fixed quantity that reflects local sample size' (i.e. still an 'adaptive' distance but the actual local sample size will be the sample size as functions are continuous). In practise a fixed bandwidth suits fairly regular sample configurations whilst an adaptive bandwidth suits highly irregular sample configurations. Adaptive bandwidths ensure sufficient (and constant) local information for each local calibration. This note is applicable to all GW models

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bw.gwpca

# Description

A function for automatic bandwidth selection to calibrate a basic or robust GWPCA via a cross-validation approach only

# Usage

# Arguments

varsa vector of variable names to be evaluatedkthe number of retained components, and it must be less than the number of variablesrobustif TRUE, robust GWPCA will be applied; otherwise basic GWPCA will be appliedscalingif TRUE, the data is scaled to have zero mean and unit variance (standardized); otherwise the data is centered but not scaledkernelfunction chosen as follows: gaussian: wgt = exp(5*(vdist/bw)^2); exponential: wgt = exp(5*(vdist/bw))^2)^2 if vdist < bw, wgt=0 otherwise; tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise; boxcar: wgt=1 if dist < bw, wgt=0 otherwise; boxcar: wgt=1 if dist < bw, wgt=0 otherwise; distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)pthe power of the Minkowski distance, default is 2, i.e. the Euclidean distancehetaan angle in radians to rotate the coordinate system, default is 0longlatif TRUE, great circle distance swill be calculateddMata pre-specified distance matrix, it can be calculated by the function gw.dist	data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>
variablesrobustif TRUE, robust GWPCA will be applied; otherwise basic GWPCA will be appliedscalingif TRUE, the data is scaled to have zero mean and unit variance (standardized); otherwise the data is centered but not scaledkernelfunction chosen as follows: gaussian: wgt = exp(5*(vdist/bw)^2); exponential: wgt = exp(-vdist/bw); bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise; tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise; boxcar: wgt=1 if dist < bw, wgt=0 otherwise	vars	a vector of variable names to be evaluated
pliedscalingif TRUE, the data is scaled to have zero mean and unit variance (standardized); otherwise the data is centered but not scaledkernelfunction chosen as follows: gaussian: wgt = exp(5*(vdist/bw)^2); exponential: wgt = exp(5*(vdist/bw))^2) (if vdist < bw, wgt=0 otherwise; tricube: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise; boxcar: wgt=1 if dist < bw, wgt=0 otherwise	k	•
kernelotherwise the data is centered but not scaledkernelfunction chosen as follows: gaussian: wgt = exp(5*(vdist/bw)^2); exponential: wgt = exp(-vdist/bw); bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise; tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise; boxcar: wgt=1 if dist < bw, wgt=0 otherwise	robust	
gaussian: wgt = exp(5*(vdist/bw)^2); exponential: wgt = exp(-vdist/bw); bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise; tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise; boxcar: wgt=1 if dist < bw, wgt=0 otherwise	scaling	
exponential: wgt = exp(-vdist/bw);bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;	kernel	function chosen as follows:
<ul> <li>bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist &lt; bw, wgt=0 otherwise; tricube: wgt = (1-(vdist/bw)^3)^3 if vdist &lt; bw, wgt=0 otherwise; boxcar: wgt=1 if dist &lt; bw, wgt=0 otherwise</li> <li>adaptive if TRUE calculate an adaptive kernel where the bandwidth corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)</li> <li>p the power of the Minkowski distance, default is 2, i.e. the Euclidean distance</li> <li>theta an angle in radians to rotate the coordinate system, default is 0</li> <li>longlat if TRUE, great circle distances will be calculated</li> </ul>		gaussian: wgt = $exp(5*(vdist/bw)^2);$
tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise; boxcar: wgt=1 if dist < bw, wgt=0 otherwiseadaptiveif TRUE calculate an adaptive kernel where the bandwidth corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)pthe power of the Minkowski distance, default is 2, i.e. the Euclidean distancethetaan angle in radians to rotate the coordinate system, default is 0longlatif TRUE, great circle distances will be calculated		exponential: wgt = exp(-vdist/bw);
boxcar: wgt=1 if dist < bw, wgt=0 otherwiseadaptiveif TRUE calculate an adaptive kernel where the bandwidth corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)pthe power of the Minkowski distance, default is 2, i.e. the Euclidean distancethetaan angle in radians to rotate the coordinate system, default is 0longlatif TRUE, great circle distances will be calculated		bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
adaptiveif TRUE calculate an adaptive kernel where the bandwidth corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)pthe power of the Minkowski distance, default is 2, i.e. the Euclidean distancethetaan angle in radians to rotate the coordinate system, default is 0longlatif TRUE, great circle distances will be calculated		tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)pthe power of the Minkowski distance, default is 2, i.e. the Euclidean distancethetaan angle in radians to rotate the coordinate system, default is 0longlatif TRUE, great circle distances will be calculated		boxcar: wgt=1 if dist < bw, wgt=0 otherwise
thetaan angle in radians to rotate the coordinate system, default is 0longlatif TRUE, great circle distances will be calculated	adaptive	number of nearest neighbours (i.e. adaptive distance); default is FALSE, where
longlat if TRUE, great circle distances will be calculated	р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
	theta	an angle in radians to rotate the coordinate system, default is 0
dMat a pre-specified distance matrix, it can be calculated by the function gw.dist	longlat	if TRUE, great circle distances will be calculated
	dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist

### Value

Returns the adaptive or fixed distance bandwidth

For a discontinuous kernel function, a bandwidth can be specified either as a fixed (constant) distance or as a fixed (constant) number of local data (i.e. an adaptive distance). For a continuous kernel function, a bandwidth can be specified either as a fixed distance or as a 'fixed quantity that reflects local sample size' (i.e. still an 'adaptive' distance but the actual local sample size will be the sample size as functions are continuous). In practise a fixed bandwidth suits fairly regular sample configurations whilst an adaptive bandwidth suits highly irregular sample configurations. Adaptive bandwidths ensure sufficient (and constant) local information for each local calibration. This note is applicable to all GW models

#### Author(s)

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#### References

Harris P, Clarke A, Juggins S, Brunsdon C, Charlton M (2015) Enhancements to a geographically weighted principal components analysis in the context of an application to an environmental data set. Geographical Analysis 47: 146-172

bw.gwr

Bandwidth selection for basic GWR

#### Description

A function for automatic bandwidth selection to calibrate a basic GWR model

# Usage

#### Arguments

formula	Regression model formula of a formula object
data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>
approach	specified by CV for cross-validation approach or by AIC corrected (AICc) approach
kernel	<pre>function chosen as follows: gaussian: wgt = exp(5*(vdist/bw)^2); exponential: wgt = exp(-vdist/bw); bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist &lt; bw, wgt=0 otherwise; tricube: wgt = (1-(vdist/bw)^3)^3 if vdist &lt; bw, wgt=0 otherwise; boxcar: wgt=1 if dist &lt; bw, wgt=0 otherwise</pre>

10

# Note

# bw.gwr.lcr

adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist
parallel.metho	d
	FALSE as default, and the calibration will be conducted traditionally via the se- rial technique, "omp": multi-thread technique with the OpenMP API, "cluster": multi-process technique with the <b>parallel</b> package, "cuda": parallel computing technique with CUDA
parallel.arg	if parallel.method is not FALSE, then set the argument by following: if parallel.method is "omp", parallel.arg refers to the number of threads used, and its default value is the number of cores - 1; if parallel.method is "cluster", parallel.arg refers to the number of R sessions used, and its default value is the number of cores - 1; if parallel.method is "cluster", parallel.arg refers to the number of cores - 1; if parallel.arg refers to the number of cores - 1; if parallel.method is "cluster", parallel.arg refers to the number of cores - 1; if parallel.method is "cluster", parallel.arg refers to the number of cores - 1; if parallel.method is "cluster", parallel.arg refers to the number of cores - 1; if parallel.method is "cluster", parallel.arg refers to the number of cores - 1; if parallel.method is "cluster", parallel.arg refers to the number of cores - 1; if parallel.method is "cluster", parallel.arg refers to the number of cores - 1; if parallel.method is "cluster", parallel.arg refers to the number of cores - 1; if parallel.method is "cluster", parallel.arg refers to the number of cores - 1; if parallel.method is "cluster", parallel.arg refers to the number of cores - 1; if parallel.method is "cluster", parallel.arg refers to the number of cores - 1; if parallel.method is "cluster", parallel.arg refers to the number of cores - 1; if parallel.method is "cluster", parallel.arg refers to the number of cores - 1; if parallel.method is "cluster", parallel.arg refers to the number of cores - 1; if parallel.method is "cluster", parallel.arg refers to the number of cores - 1; if parallel.method is "cluster", parallel.arg refers to the number of cores - 1; if parallel.method is "cluster", parallel.arg refers to the number of cluster.

#### Value

Returns the adaptive or fixed distance bandwidth

### Note

For a discontinuous kernel function, a bandwidth can be specified either as a fixed (constant) distance or as a fixed (constant) number of local data (i.e. an adaptive distance). For a continuous kernel function, a bandwidth can be specified either as a fixed distance or as a 'fixed quantity that reflects local sample size' (i.e. still an 'adaptive' distance but the actual local sample size will be the sample size as functions are continuous). In practise a fixed bandwidth suits fairly regular sample configurations whilst an adaptive bandwidth suits highly irregular sample configurations. Adaptive bandwidths ensure sufficient (and constant) local information for each local calibration. This note is applicable to all GW models

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bw.gwr.lcr

Bandwidth selection for locally compensated ridge GWR (GWR-LCR)

# Description

A function for automatic bandwidth selection for gwr.lcr via a cross-validation approach only

# Usage

# Arguments

formula	Regression model formula of a formula object
data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>
kernel	function chosen as follows:
	gaussian: wgt = exp(5*(vdist/bw)^2); exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = $(1-(vdist/bw)^2)^2$ if vdist < bw, wgt=0 otherwise;
	tricube: wgt = $(1 - (vdist/bw)^2)^2$ if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
lambda	option for a globally-defined (constant) ridge parameter. Default is lambda=0, which gives a basic GWR fit
lambda.adjust	a locally-varying ridge parameter. Default FALSE, refers to: (i) a basic GWR without a local ridge adjustment (i.e. lambda=0, everywhere); or (ii) a penalised GWR with a global ridge adjustment (i.e. lambda is user-specified as some constant, other than 0 everywhere); if TRUE, use cn.tresh to set the maximum condition number. For locations with a condition number (for its local design matrix), above this user-specified threshold, a local ridge parameter is found
cn.thresh	maximum value for condition number, commonly set between 20 and 30
adaptive	if TRUE calculate an adaptive kernel where the bandwidth corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist

#### Value

Returns the adaptive or fixed distance bandwidth

### Note

For a discontinuous kernel function, a bandwidth can be specified either as a fixed (constant) distance or as a fixed (constant) number of local data (i.e. an adaptive distance). For a continuous kernel function, a bandwidth can be specified either as a fixed distance or as a 'fixed quantity that reflects local sample size' (i.e. still an 'adaptive' distance but the actual local sample size will be the sample size as functions are continuous). In practise a fixed bandwidth suits fairly regular sample configurations whilst an adaptive bandwidth suits highly irregular sample configurations. Adaptive bandwidths ensure sufficient (and constant) local information for each local calibration. This note is applicable to all GW models

# bw.gwss.average

### Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

# References

Gollini I, Lu B, Charlton M, Brunsdon C, Harris P (2015) GWmodel: an R Package for exploring Spatial Heterogeneity using Geographically Weighted Models. Journal of Statistical Software 63(17): 1-50

bw.gwss.average Bandwidth

# Bandwidth selection for GW summary averages

# Description

A function for automatic bandwidth selections to calculate GW summary averages, including means and medians, via a cross-validation approach.

# Usage

# Arguments

data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b>
summary.locat	a Spatial*DataFrame object for providing summary locations, i.e. SpatialPoints- DataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b>
vars	a vector of variable names to be summarized
kernel	function chosen as follows:
	gaussian: wgt = $exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
	tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist

#### Value

Returns the adaptive or fixed distance bandwidths (in a two-column matrix) for calculating the averages of each variable.

### Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

DubVoter

*Voter turnout data in Greater Dublin(SpatialPolygonsDataFrame)* 

#### Description

Voter turnout and social characters data in Greater Dublin for the 2002 General election and the 2002 census. Note that this data set was originally thought to relate to 2004, so for continuity we have retained the associated variable names.

## Usage

data(DubVoter)

#### Format

A SpatialPolygonsDataFrame with 322 electoral divisions on the following 11 variables.

**DED\_ID** a vector of ID

X a numeric vector of x coordinates

Y a numeric vector of y coordinates

**DiffAdd** percentage of the population in each ED who are one-year migrants (i.e. moved to a different address 1 year ago)

LARent percentage of the population in each ED who are local authority renters

SC1 percentage of the population in each ED who are social class one (high social class)

Unempl percentage of the population in each ED who are unemployed

LowEduc percentage of the population in each ED who are with little formal education

Age18\_24 percentage of the population in each ED who are age group 18-24

Age25\_44 percentage of the population in each ED who are age group 25-44

Age45\_64 percentage of the population in each ED who are age group 45-64

GenEl2004 percentage of population in each ED who voted in 2004 election

# Details

Variables are from DubVoter.shp.

### EWHP

## References

Kavanagh A (2006) Turnout or turned off? Electoral participation in Dublin in the early 21st Century. Journal of Irish Urban Studies 3(2):1-24

Harris P, Brunsdon C, Charlton M (2011) Geographically weighted principal components analysis. International Journal of Geographical Information Science 25 (10):1717-1736

# Examples

```
data(DubVoter)
ls()
## Not run:
spplot(Dub.voter,names(Dub.voter)[4:12])
```

## End(Not run)

EWHP

House price data set (DataFrame) in England and Wales

#### Description

A house price data set for England and Wales from 2001 with 9 hedonic (explanatory) variables.

#### Usage

data(EWHP)

#### Format

A data frame with 519 observations on the following 12 variables.

Easting a numeric vector, X coordinate

Northing a numeric vector, Y coordinate

PurPrice a numeric vector, the purchase price of the property

BldIntWr a numeric vector, 1 if the property was built during the world war, 0 otherwise

**BidPostW** a numeric vector, 1 if the property was built after the world war, 0 otherwise

Bld60s a numeric vector, 1 if the property was built between 1960 and 1969, 0 otherwise

Bld70s a numeric vector, 1 if the property was built between 1970 and 1979, 0 otherwise

Bld80s a numeric vector, 1 if the property was built between 1980 and 1989, 0 otherwise

TypDetch a numeric vector, 1 if the property is detached (i.e. it is a stand-alone house), 0 otherwise

TypSemiD a numeric vector, 1 if the property is semi detached, 0 otherwise

TypFlat a numeric vector, if the property is a flat (or 'apartment' in the USA), 0 otherwise

FlrArea a numeric vector, floor area of the property in square metres

### Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

### References

Fotheringham, A.S., Brunsdon, C., and Charlton, M.E. (2002), Geographically Weighted Regression: The Analysis of Spatially Varying Relationships, Chichester: Wiley.

#### Examples

```
###
data(EWHP)
head(ewhp)
houses.spdf <- SpatialPointsDataFrame(ewhp[, 1:2], ewhp)
####Get the border of England and Wales
data(EWOutline)
plot(ewoutline)
plot(houses.spdf, add = TRUE, pch = 16)</pre>
```

EWOutline

Outline of England and Wales for data EWHP

### Description

Outline (SpatialPolygonsDataFrame) of the England and Wales house price data EWHP.

#### Usage

data(EWOutline)

#### Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

Georgia

Georgia census data set (csv file)

# Description

Census data from the county of Georgia, USA

### Usage

data(Georgia)

# GeorgiaCounties

#### Format

A data frame with 159 observations on the following 13 variables.

AreaKey An identification number for each county

Latitude The latitude of the county centroid

Longitud The longitude of the county centroid

TotPop90 Population of the county in 1990

PctRural Percentage of the county population defined as rural

PctBach Percentage of the county population with a bachelors degree

PctEld Percentage of the county population aged 65 or over

PctFB Percentage of the county population born outside the US

PctPov Percentage of the county population living below the poverty line

PctBlack Percentage of the county population who are black

ID a numeric vector of IDs

X a numeric vector of x coordinates

Y a numeric vector of y coordinates

#### Details

This data set can also be found in GWR 3 and in spgwr.

#### References

Fotheringham S, Brunsdon, C, and Charlton, M (2002), Geographically Weighted Regression: The Analysis of Spatially Varying Relationships, Chichester: Wiley.

### Examples

```
data(Georgia)
ls()
coords <- cbind(Gedu.df$X, Gedu.df$Y)
educ.spdf <- SpatialPointsDataFrame(coords, Gedu.df)
spplot(educ.spdf, names(educ.spdf)[4:10])</pre>
```

GeorgiaCounties Georgia counties data (SpatialPolygonsDataFrame)

# Description

The Georgia census data with boundaries for mapping

#### Usage

data(GeorgiaCounties)

# Details

This data set can also be found in GWR 3 and in spgwr.

# Examples

```
data(GeorgiaCounties)
plot(Gedu.counties)
data(Georgia)
coords <- cbind(Gedu.df$X, Gedu.df$Y)
educ.spdf <- SpatialPointsDataFrame(coords, Gedu.df)
plot(educ.spdf, add=TRUE)</pre>
```

ggwr.basic

Generalised GWR models with Poisson and Binomial options

# Description

This function implements generalised GWR

# Usage

```
ggwr.basic(formula, data, regression.points, bw, family =
    "poisson", kernel = "bisquare", adaptive = FALSE, cv =
    T, tol = 1e-05, maxiter = 20, p = 2, theta = 0,
    longlat = F, dMat, dMat1)
```

## S3 method for class 'ggwrm'
print(x, ...)

#### Arguments

formula	Regression model formula of a formula object
data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>
regression.points	
	a Spatial*DataFrame object, i.e. SpatialPointsDataFrame or SpatialPolygons-
	DataFrame as defined in package <b>sp</b>
bw	bandwidth used in the weighting function, possibly calculated by bw.ggwr();fixed
	(distance) or adaptive bandwidth(number of nearest neighbours)
family	a description of the error distribution and link function to be used in the model, which can be specified by "poisson" or "binomial"

18

# ggwr.basic

kernel	function chosen as follows:
	gaussian: wgt = $\exp(5*(vdist/bw)^2)$ ;
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
	tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
adaptive	if TRUE calculate an adaptive kernel where the bandwidth corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
CV	if TRUE, cross-validation data will be calculated
tol	the threshold that determines the convergence of the IRLS procedure
maxiter	the maximum number of times to try the IRLS procedure
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix between regression points and observations, it can be calculated by the function gw.dist
dMat1	a square distance matrix between each pair of observations, it can be calculated by the function $gw.dist$
х	an object of class "ggwrm", returned by the function gwr.generalised
	arguments passed through (unused)

# Value

A list of class "ggwrm":

GW.arguments	a list class object including the model fitting parameters for generating the report file
GW.diagnostic	a list class object including the diagnostic information of the model fitting
glm.res	an object of class inheriting from "glm" which inherits from the class "lm", see glm.
SDF	a SpatialPointsDataFrame (may be gridded), or SpatialPolygonsDataFrame object (see package "sp"), or sf object (see package "sf") integrated with regression.points, GWR coefficient estimates, y value,predicted values, coefficient standard errors and t-values in its "data" slot.
CV	a data vector consisting of the cross-validation data

# Note

Note that this function calibrates a Generalised GWR model via an approximating algorithm, which is different from the back-fitting algorithm used in the GWR4 software by Tomoki Nakaya.

# Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

#### References

Nakaya, T., A. S. Fotheringham, C. Brunsdon & M. Charlton (2005) Geographically weighted Poisson regression for disease association mapping. Statistics in Medicine, 24, 2695-2717.

Nakaya, T., M. Charlton, S. Fotheringham & C. Brunsdon. 2009. How to use SGWRWIN (GWR4.0). Maynooth, Ireland: National Centre for Geocomputation.

Fotheringham S, Brunsdon, C, and Charlton, M (2002), Geographically Weighted Regression: The Analysis of Spatially Varying Relationships, Chichester: Wiley.

#### Examples

## End(Not run)

ggwr.cv

Cross-validation score for a specified bandwidth for generalised GWR

#### Description

This function finds the cross-validation score for a specified bandwidth for generalised GWR. It can be used to construct the bandwidth function across all possible bandwidths and compared to that found automatically.

# Usage

#### Arguments

bw	bandwidth used in the weighting function; fixed (distance) or adaptive band-width(number of nearest neighbours)
Х	a numeric matrix of the independent data with an extra column of "ones" for the 1st column
Υ	a column vector of the dependent data
family	a description of the error distribution and link function to be used in the model, which can be specified by "poisson" or "binomial"

20

# ggwr.cv.contrib

kernel	function chosen as follows:
	gaussian: wgt = $exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
	tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
dp.locat	a two-column numeric array of observation coordinates
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist

# Value

cv.score cross-vandation score	CV.score	cross-validation score
--------------------------------	----------	------------------------

# Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

ggwr.cv.contrib	Cross-validation data at each observation location for a generalised GWR model

# Description

This function finds the individual cross-validation score at each observation location, for a generalised GWR model, for a specified bandwidth. These data can be mapped to detect unusually high or low cross-validations scores.

# Usage

# Arguments

bw	bandwidth used in the weighting function; fixed (distance) or adaptive band- width(number of nearest neighbours)
Х	a numeric matrix of the independent data with an extra column of "ones" for the 1st column
Υ	a column vector of the dependent data

family	a description of the error distribution and link function to be used in the model, which can be specified by "poisson" or "binomial"
kernel	function chosen as follows:
	gaussian: wgt = $exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
	tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
dp.locat	a two-column numeric array of observation coordinates
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist

# Value

CV	a data vector consisting of squared residuals, whose sum is the cross-validation
	score for the specified bandwidth

# Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

gtwr

Geographically and Temporally Weighted Regression

# Description

A function for calibrating a Geographically and Temporally Weighted Regression (GTWR) model.

# Usage

# gtwr

# Arguments

formula	Regression model formula of a formula object
data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>
regression.poir	nts
	a Spatial*DataFrame object, i.e. SpatialPointsDataFrame or SpatialPolygons- DataFrame as defined in package <b>sp</b> ; Note that no diagnostic information will returned if it is assigned
obs.tv	a vector of time tags for each observation, which could be numeric or of <b>POSIXI</b> t class
reg.tv	a vector of time tags for each regression location, which could be numeric or of <b>POSIXIt</b> class
st.bw	spatio-temporal bandwidth used in the weighting function, possibly calculated by bw.gwr;fixed (distance) or adaptive bandwidth(number of nearest neighbours)
kernel	function chosen as follows:
	gaussian: $wgt = exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
	tricube: $wgt = (1-(vdist/bw)^3)^3$ if $vdist < bw, wgt=0$ otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
lamda	an parameter between 0 and 1 for calculating spatio-temporal distance
t.units	character string to define time unit
ksi	an parameter between 0 and PI for calculating spatio-temporal distance, see de- tails in Wu et al. (2014)
st.dMat	a pre-specified spatio-temporal distance matrix, and can be calculated via the function st.dist

# Value

A list of class "gtwrm":

GTW.arguments	a list class object including the model fitting parameters for generating the report file
GTW.diagnostic	a list class object including the diagnostic information of the model fitting
lm	an object of class inheriting from "lm", see lm.

SDF	a SpatialPointsDataFrame (may be gridded), or SpatialPolygonsDataFrame object (see package "sp"), or sf object (see package "sf") integrated with regression.points, GTWR coefficient estimates, y value,predicted values, coefficient standard errors and t-values in its "data" slot.
timings	starting and ending time.
this.call	the function call used.

# Note

The function implements GTWR model proposed by Huang et al. (2010) and Wu et al. (2014).

### Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

### References

Huang, B., Wu, B., & Barry, M. (2010). Geographically and temporally weighted regression for modeling spatio-temporal variation in house prices. International Journal of Geographical Information Science, 24, 383-401.

Wu, B., Li, R., & Huang, B. (2014). A geographically and temporally weighted autoregressive model with application to housing prices. International Journal of Geographical Information Science, 28, 1186-1204.

Fotheringham, A. S., Crespo, R., & Yao, J. (2015). Geographical and Temporal Weighted Regression (GTWR). Geographical Analysis, 47, 431-452.

# Description

Calculate a distance vector(matrix) between any GW model calibration point(s) and the data points.

# Usage

```
gw.dist(dp.locat, rp.locat, focus=0, p=2, theta=0, longlat=F)
```

#### Arguments

dp.locat	a numeric matrix of two columns giving the coordinates of the data points
rp.locat	a numeric matrix of two columns giving the coordinates of the GW model calibration points
focus	an integer, indexing to the current GW model point, if focus=0, all the distances between all the GW model calibration points and data points will be calculated and a distance matrix will be returned; if 0 <focus<length(rp.locat), 'focus'th="" a="" and="" be="" between="" calculated="" data="" distance="" distances="" gw="" model="" points="" returned<="" td="" the="" then="" vector="" will=""></focus<length(rp.locat),>

#### gw.dist

р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated

#### Value

Returns a numeric distance matrix or vector; matrix with its rows corresponding to the observations and its columns corresponds to the GW model calibration points.

### Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

# See Also

dist in stats

#### Examples

```
dp<-cbind(sample(100), sample(100))</pre>
rp<-cbind(sample(10), sample(10))</pre>
#Euclidean distance metric is used.
dist.v1<-gw.dist(dp.locat=dp, focus=5, p=2, theta=0, longlat=FALSE)</pre>
#Manhattan distance metric is used.
#The coordinate system is rotated by an angle 0.5 in radian.
dist.v2<-gw.dist(dp.locat=dp, focus=5, p=1, theta=0.5)</pre>
#Great Circle distance metric is used.
dist.v3<-gw.dist(dp.locat=dp, focus=5, longlat=TRUE)</pre>
#A generalized Minkowski distance metric is used with p= 0.75 .
#The coordinate system is rotated by an angle 0.8 in radian.
dist.v4<-gw.dist(dp.locat=dp,rp.locat=rp, focus=5, p=0.75,theta=0.8)</pre>
#matrix is calculated
#Euclidean distance metric is used.
dist.m1<-gw.dist(dp.locat=dp, p=2, theta=0, longlat=FALSE)
#Manhattan distance metric is used.
#The coordinate system is rotated by an angle 0.5 in radian.
dist.m2<-gw.dist(dp.locat=dp, p=1, theta=0.5)</pre>
#Great Circle distance metric is used.
#dist.m3<-gw.dist(dp.locat=dp, longlat=TRUE)</pre>
#A generalized Minkowski distance metric is used with p= 0.75 .
#The coordinate system is rotated by an angle 0.8 in radian.
dist.m4<-gw.dist(dp.locat=dp,rp.locat=rp, p=0.75,theta=0.8)</pre>
```

gw.pcplot

# Description

This function provides a geographically weighted parallel coordinate plot for locally investigating a multivariate data set. It has an option that weights the lines of the plot with increasing levels of transparency, according to their observation's distance from a specified focal/observation point.

### Usage

# Arguments

data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>
vars	a vector of variable names to be evaluated
focus	an integer, indexing to the observation point
bw	bandwidth used in the weighting function; fixed (distance) or adaptive band-width(number of nearest neighbours)
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
ylim	the y limits of the plot
ylab	a label for the y axis
fixtrans	if TRUE, the transparency of the neighbouring observation plot lines increases with distance; If FALSE a standard (non-spatial) parallel coordinate plot is re- turned.
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist
	other graphical parameters, (see par)

# Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

#### gw.weight

### References

Harris P, Brunsdon C, Charlton M, Juggins S, Clarke A (2014) Multivariate spatial outlier detection using robust geographically weighted methods. Mathematical Geosciences 46(1) 1-31

Harris P, Clarke A, Juggins S, Brunsdon C, Charlton M (2015) Enhancements to a geographically weighted principal components analysis in the context of an application to an environmental data set. Geographical Analysis 47: 146-172

```
gw.weight
```

Weight matrix calculation

#### Description

Calculate a weight vector(matrix) from a distance vector(matrix).

#### Usage

gw.weight(vdist,bw,kernel,adaptive=FALSE)

#### Arguments

vdist	a distance matrix or vector
bw	bandwidth used in the weighting function, possibly calculated by bw.gwr;fixed (distance) or adaptive bandwidth(number of nearest neighbours)
kernel	function chosen as follows:
	gaussian: wgt = $exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
	tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)

#### Value

Returns a numeric weight matrix or vector; matrix with its rows corresponding to the observations and its columns corresponds to the GW model calibration points.

#### Note

The gaussian and exponential kernel functions are continuous and valued in the interval (0,1]; while bisquare, tricube and boxcar kernel functions are discontinuous and valued in the interval [0,1]. Notably, the upper limit of the bandwidth is exactly the number of observations when the adaptive kernel is used. In this function, the adaptive bandwidth will be specified as the number of observations even though a larger number is assigned. The function will be the same as a global application function (i.e. all weights are 1) when the adaptive bandwidth is equal to or larger than the number of observations when using the boxcar kernel function.

## Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

gwda

GW Discriminant Analysis

# Description

This function implements GW discriminant analysis, where location-wise probabilities and their associated entropy are also calculated.

# Usage

# Arguments

formula	Model formula of a formula object
data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>
predict.data	a Spatial*DataFrame object for prediction, i.e. SpatialPointsDataFrame or Spa- tialPolygonsDataFrame as defined in package <b>sp</b> ; if it is not given, the traing data will be predicted using leave-one-out cross-validation.
validation	If TRUE, the results from the prediction will be validated and the correct pro- portion will be calculated.
COV.gw	if true, localised variance-covariance matrix is used for GW discriminant analy- sis; otherwise, global variance-covariance matrix is used
mean.gw	if true, localised mean is used for GW discriminant analysis; otherwise, global mean is used
prior.gw	if true, localised prior probability is used for GW discriminant analysis; other- wise, fixed prior probability is used
prior	a vector of given prior probability
wqda	if TRUE, weighted quadratic discriminant analysis will be applied; otherwise weighted linear discriminant analysis will be applied
kernel	function chosen as follows:
	gaussian: $wgt = exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = $(1-(vdist/bw)^2)^2$ if vdist < bw, wgt=0 otherwise;
	tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise

#### gwda

adaptive	if TRUE calculate an adaptive kernel where the bandwidth corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
bw	bandwidth used in the weighting function, possibly calculated by bw.gwpca;fixed (distance) or adaptive bandwidth(number of nearest neighbours)
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist
x	an object of class "gwda"
	arguments passed through (unused)

# Value

An object of class "gwda". This includes a SpatialPointsDataFrame (may be gridded) or SpatialPolygonsDataFrame object, SDF, (see package "sp") or sf object (see package "sf") with, following the use of new version of gwda, the probabilities for each level, the highest probability and the entropy of the probabilities in its "data" slot.

### Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

# References

Brunsdon, C, Fotheringham S, and Charlton, M (2007), Geographically Weighted Discriminant Analysis, Geographical Analysis 39:376-396

Lu B, Harris P, Charlton M, Brunsdon C (2014) The GWmodel R Package: further topics for exploring Spatial Heterogeneity using Geographically Weighted Models. Geo-spatial Information Science 17(2): 85-101

### Examples

```
## Not run:
data(USelect)
dMat <- gw.dist(coordinates(USelect2004))
bw <- bw.gwda(winner~unemploy+pctcoled+PEROVER65+pcturban+WHITE,data=USelect2004,
adaptive=TRUE,dMat=dMat)
ge.gwda <- gwda(winner~unemploy+pctcoled+PEROVER65+pcturban+WHITE,data=USelect2004,
bw=bw,adaptive=TRUE,dMat=dMat)
table(USelect2004$winner,ge.gwda$SDF$group.predicted)
spplot(ge.gwda$SDF, "entropy")
```

## End(Not run)

gwpca

# Description

This function implements basic or robust GWPCA.

# Usage

# Arguments

data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>
elocat	a two-column numeric array or Spatial*DataFrame object for providing evalu- ation locations, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b>
vars	a vector of variable names to be evaluated
k	the number of retained components; k must be less than the number of variables
robust	if TRUE, robust GWPCA will be applied; otherwise basic GWPCA will be applied
scaling	if TRUE, the data is scaled to have zero mean and unit variance (standardized); otherwise the data is centered but not scaled
kernel	<pre>function chosen as follows: gaussian: wgt = exp(5*(vdist/bw)^2); exponential: wgt = exp(-vdist/bw); bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist &lt; bw, wgt=0 otherwise; tricube: wgt = (1-(vdist/bw)^3)^3 if vdist &lt; bw, wgt=0 otherwise; boxcar: wgt=1 if dist &lt; bw, wgt=0 otherwise</pre>
adaptive	if TRUE calculate an adaptive kernel where the bandwidth corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
bw	bandwidth used in the weighting function, possibly calculated by bw.gwpca;fixed (distance) or adaptive bandwidth(number of nearest neighbours)
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated

#### gwpca

cv	If TRUE, cross-validation data will be found that are used to calculate the cross-validation score for the specified bandwidth.
scores	if scores = TRUE, the scores of the supplied data on the principal components will be calculated.
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist
x	an object of class "gwpca", returned by the function gwpca
	arguments passed through (unused)

### Value

A list of class "gwpca":

GW.arguments	a list class object including the model fitting parameters for generating the report file
рса	an object of class inheriting from "princomp", see princomp.
loadings	the localised loadings
SDF	a SpatialPointsDataFrame (may be gridded) or SpatialPolygonsDataFrame object (see package "sp") or sf object (see package "sf") integrated with local proportions of variance for each principle components, cumulative proportion and winning variable for the 1st principle component in its "data" slot.
gwpca.scores	the localised scores of the supplied data on the principal components
var	The local amount of variance accounted for by each component
CV	Vector of cross-validation data
timings	starting and ending time.

# Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

#### References

Fotheringham S, Brunsdon, C, and Charlton, M (2002), Geographically Weighted Regression: The Analysis of Spatially Varying Relationships, Chichester: Wiley.

Harris P, Brunsdon C, Charlton M (2011) Geographically weighted principal components analysis. International Journal of Geographical Information Science 25:1717-1736

Harris P, Brunsdon C, Charlton M, Juggins S, Clarke A (2014) Multivariate spatial outlier detection using robust geographically weighted methods. Mathematical Geosciences 46(1) 1-31

Harris P, Clarke A, Juggins S, Brunsdon C, Charlton M (2014) Geographically weighted methods and their use in network re-designs for environmental monitoring. Stochastic Environmental Research and Risk Assessment 28: 1869-1887

Harris P, Clarke A, Juggins S, Brunsdon C, Charlton M (2015) Enhancements to a geographically weighted principal components analysis in the context of an application to an environmental data set. Geographical Analysis 47: 146-172

# Examples

{

```
## Not run:
if(require("mvoutlier") && require("RColorBrewer"))
 data(bsstop)
 Data.1 <- bsstop[, 1:14]</pre>
 colnames(Data.1)
 Data.1.scaled <- scale(as.matrix(Data.1[5:14])) # standardised data...</pre>
 rownames(Data.1.scaled) <- Data.1[, 1]</pre>
 #compute principal components:
 pca <- princomp(Data.1.scaled, cor = FALSE, scores = TRUE)</pre>
 # use covariance matrix to match the following...
 pca$loadings
 data(bss.background)
 backdrop <- function()</pre>
  plot(bss.background, asp = 1, type = "l", xaxt = "n", yaxt = "n",
  xlab = "", ylab = "", bty = "n", col = "grey")
 pc1 <- pca$scores[, 1]</pre>
 backdrop()
 points(Data.1$XC00[pc1 > 0], Data.1$YC00[pc1 > 0], pch = 16, col = "blue")
 points(Data.1$XC00[pc1 < 0], Data.1$YC00[pc1 < 0], pch = 16, col = "red")</pre>
 #Geographically Weighted PCA and mapping the local loadings
 # Coordinates of the sites
 Coords1 <- as.matrix(cbind(Data.1$XC00,Data.1$YC00))</pre>
 d1s <- SpatialPointsDataFrame(Coords1,as.data.frame(Data.1.scaled))
 pca.gw <- gwpca(d1s,vars=colnames(d1s@data),bw=1000000,k=10)</pre>
 local.loadings <- pca.gw$loadings[, , 1]</pre>
 # Mapping the winning variable with the highest absolute loading
 # note first component only - would need to explore all components..
 lead.item <- colnames(local.loadings)[max.col(abs(local.loadings))]</pre>
 df1p = SpatialPointsDataFrame(Coords1, data.frame(lead = lead.item))
 backdrop()
 colour <- brewer.pal(8, "Dark2")[match(df1p$lead, unique(df1p$lead))]</pre>
 plot(df1p, pch = 18, col = colour, add = TRUE)
 legend("topleft", as.character(unique(df1p$lead)), pch = 18, col =
      brewer.pal(8, "Dark2"))
 backdrop()
 #Glyph plots give a view of all the local loadings together
 glyph.plot(local.loadings, Coords1, add = TRUE)
 #it is not immediately clear how to interpret the glyphs fully,
 #so inter-actively identify the full loading information using:
 check.components(local.loadings, Coords1)
 # GWPCA with an optimal bandwidth
 bw.choice <- bw.gwpca(d1s,vars=colnames(d1s@data),k=2)</pre>
 pca.gw.auto <- gwpca(d1s,vars=colnames(d1s@data),bw=bw.choice,k=2)</pre>
 # note first component only - would need to explore all components..
```

32

```
local.loadings <- pca.gw.auto$loadings[, , 1]
lead.item <- colnames(local.loadings)[max.col(abs(local.loadings))]
df1p = SpatialPointsDataFrame(Coords1, data.frame(lead = lead.item))
backdrop()
colour <- brewer.pal(8, "Dark2")[match(df1p$lead, unique(df1p$lead))]
plot(df1p, pch = 18, col = colour, add = TRUE)
legend("topleft", as.character(unique(df1p$lead)), pch = 18,
col = brewer.pal(8, "Dark2"))
# GWPCPLOT for investigating the raw multivariate data
gw.pcplot(d1s, vars=colnames(d1s@data),focus=359, bw = bw.choice)
}
## End(Not run)
```

gwpca.check.components

Interaction tool with the GWPCA glyph map

# Description

The function interacts with the multivariate glyph plot of GWPCA loadings.

#### Usage

```
gwpca.check.components(ld,loc)
```

#### Arguments

ld	GWPCA loadings returned by gwpca
loc	a 2-column numeric array of GWPCA evaluation locations

# Note

The function "check.components" (in the early versions of GWmodel) has been renamed as "gw-pca.check.components", while the old name is still kept valid.

# Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

#### See Also

gwpca.glyph.plot

gwpca.cv

# Description

This function finds the cross-validation score for a specified bandwidth for basic or robust GWPCA. It can be used to construct the bandwidth function across all possible bandwidths and compared to that found automatically.

# Usage

### Arguments

bw	bandwidth used in the weighting function; fixed (distance) or adaptive band- width(number of nearest neighbours)
х	the variable matrix
loc	a two-column numeric array of observation coordinates
k	the number of retained components; k must be less than the number of variables
robust	if TRUE, robust GWPCA will be applied; otherwise basic GWPCA will be applied
kernel	function chosen as follows:
	gaussian: $wgt = exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
	tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist

# Value

CV. score cross-validation score

# Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

gwpca.cv.contrib

# Description

This function finds the individual cross-validation score at each observation location, for a GWPCA model, for a specified bandwidth. These data can be mapped to detect unusually high or low cross-validations scores.

# Usage

# Arguments

х	the variable matrix
loc	a two-column numeric array of observation coordinates
bw	bandwidth used in the weighting function; fixed (distance) or adaptive band- width(number of nearest neighbours)
k	the number of retained components; k must be less than the number of variables
robust	if TRUE, robust GWPCA will be applied; otherwise basic GWPCA will be applied
kernel	function chosen as follows:
	gaussian: wgt = $\exp(5*(vdist/bw)^2)$ ;
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
	tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist
alue	

# Value

CV	a data vector consisting of squared residuals, whose sum is the cross-validation
	score for the specified bandwidth (bw) and component (k).

#### Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

gwpca.glyph.plot

#### Description

This function provides a multivariate glyph plot of GWPCA loadings at each output location.

# Usage

gwpca.glyph.plot(ld,loc, r1=50, add=FALSE,alpha=1,sep.contrasts=FALSE)

### Arguments

ld	GWPCA loadings returned by gwpca
loc	a two-column numeric array for providing evaluation locations of GWPCA cal- ibration
r1	argument for the size of the glyphs, default is 50; glyphs get larger as r1 is reduced
add	if TRUE, add the plot to the existing window.
alpha	the level of transparency of glyph from function rgb() and ranges from 0 to max (fully transparent to opaque)
sep.contrasts	allows different types of glyphs and relates to whether absolute loadings are used (TRUE) or not

#### Note

The function "glyph.plot" (in the early versions of GWmodel) has been renamed as "gwpca.glyph.plot", while the old name is still kept valid.

### References

Harris P, Brunsdon C, Charlton M (2011) Geographically weighted principal components analysis. International Journal of Geographical Information Science 25:1717-1736

gwpca.montecarlo.1	Monte Carlo (randomisation) test for significance of GWPCA eigen-
	value variability for the first component only - option 1

# Description

This function implements a Monte Carlo (randomisation) test for a basic or robust GW PCA with the bandwidth pre-specified and constant. The test evaluates whether the GW eigenvalues vary significantly across space for the first component only.
# Usage

# Arguments

data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b>
bw	bandwidth used in the weighting function, possibly calculated by bw.gwpca;fixed (distance) or adaptive bandwidth(number of nearest neighbours)
vars	a vector of variable names to be evaluated
k	the number of retained components; k must be less than the number of variables
nsims	the number of simulations for MontCarlo test
robust	if TRUE, robust GWPCA will be applied; otherwise basic GWPCA will be applied
scaling	if TRUE, the data is scaled to have zero mean and unit variance (standardized); otherwise the data is centered but not scaled
kernel	function chosen as follows:
	gaussian: $wgt = exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = $(1-(vdist/bw)^2)^2$ if vdist < bw, wgt=0 otherwise;
	tricube: $wgt = (1-(vdist/bw)^3)^3$ if $vdist < bw$ , $wgt=0$ otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist
x	an object of class "mcsims", returned by the function gwpca.montecarlo.1 or gwpca.montecarlo.2
sname	the label for the observed value on the plot
	arguments passed through (unused)

## Value

A list of components:

actual	the observed standard deviations (SD) of eigenvalues
sims	a vector of the simulated SDs of eigenvalues

The function "montecarlo.gwpca.1" (in the early versions of GWmodel) has been renamed as "gwpca.montecarlo.1", while the old name is still kept valid.

## Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

### References

Harris P, Brunsdon C, Charlton M (2011) Geographically weighted principal components analysis. International Journal of Geographical Information Science 25:1717-1736

#### Examples

```
## Not run:
data(DubVoter)
DM<-gw.dist(dp.locat=coordinates(Dub.voter))
gmc.res<-gwpca.montecarlo.1(data=Dub.voter, vars=c("DiffAdd", "LARent",
"SC1", "Unempl", "LowEduc"), bw=20,dMat=DM,adaptive=TRUE)
gmc.res
plot(gmc.res)
## End(Not run)
```

gwpca.montecarlo.2	Monte Carlo (randomisation) test for significance of GWPCA eigen-
	value variability for the first component only - option 2

### Description

This function implements a Monte Carlo (randomisation) test for a basic or robust GW PCA with the bandwidth automatically re-selected via the cross-validation approach. The test evaluates whether the GW eigenvalues vary significantly across space for the first component only.

## Usage

## Arguments

data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b>
vars	a vector of variable names to be evaluated
k	the number of retained components; k must be less than the number of variables

38

nsims	the number of simulations for MontCarlo test
robust	if TRUE, robust GWPCA will be applied; otherwise basic GWPCA will be applied
scaling	if TRUE, the data is scaled to have zero mean and unit variance (standardized); otherwise the data is centered but not scaled
kernel	function chosen as follows:
	gaussian: wgt = $exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
	tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist

#### Value

A list of components:

actual	the observed standard deviations (SD) of eigenvalues
sims	a vector of the simulated SDs of eigenvalues

## Note

The function "montecarlo.gwpca.2" (in the early versions of GWmodel) has been renamed as "gw-pca.montecarlo.2", while the old name is still kept valid.

## Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

## References

Harris P, Brunsdon C, Charlton M (2011) Geographically weighted principal components analysis. International Journal of Geographical Information Science 25:1717-1736

## Examples

```
## Not run:
data(DubVoter)
DM<-gw.dist(dp.locat=coordinates(Dub.voter))
gmc.res.autow<-gwpca.montecarlo.2(data=Dub.voter, vars=c("DiffAdd", "LARent",
    "SC1", "Unempl", "LowEduc"), dMat=DM,adaptive=TRUE)
```

gwr.basic

```
gmc.res.autow
plot.mcsims(gmc.res.autow)
## End(Not run)
```

gwr.basic

Basic GWR model

# Description

This function implements basic GWR

## Usage

```
gwr.basic(formula, data, regression.points, bw, kernel="bisquare",
adaptive=FALSE, p=2, theta=0, longlat=F,dMat,F123.test=F,cv=F, W.vect=NULL,
parallel.method=FALSE,parallel.arg=NULL)
## S3 method for class 'gwrm'
print(x, ...)
```

#### Arguments

Regression model formula of a formula object
a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>
its
a Spatial*DataFrame object, i.e. SpatialPointsDataFrame or SpatialPolygons- DataFrame as defined in package <b>sp</b> ; Note that no diagnostic information will returned if it is assigned
bandwidth used in the weighting function, possibly calculated by bw.gwr;fixed (distance) or adaptive bandwidth(number of nearest neighbours)
function chosen as follows:
gaussian: $wgt = exp(5*(vdist/bw)^2);$
exponential: wgt = exp(-vdist/bw);
bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
boxcar: wgt=1 if dist < bw, wgt=0 otherwise
if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
an angle in radians to rotate the coordinate system, default is 0
if TRUE, great circle distances will be calculated
a pre-specified distance matrix, it can be calculated by the function gw.dist

40

## gwr.basic

F123.test	If TRUE, conduct three seperate F-tests according to Leung et al. (2000).
CV	if TRUE, cross-validation data will be calculated and returned in the output Spa- tial*DataFrame
W.vect	default NULL, if given it will be used to weight the distance weighting matrix
x	an object of class "gwrm", returned by the function gwr.basic
parallel.method	d
	FALSE as default, and the calibration will be conducted traditionally via the se- rial technique, "omp": multi-thread technique with the OpenMP API, "cluster": multi-process technique with the <b>parallel</b> package, "cuda": parallel computing technique with CUDA
parallel.arg	if parallel.method is not FALSE, then set the argument by following: if parallel.method is "omp", parallel.arg refers to the number of threads used, and its default value is the number of cores - 1; if parallel.method is "cluster", parallel.arg refers to the number of R sessions used, and its default value is the number of cores - 1; if parallel.method is "cuda", parallel.arg refers to the number of calibrations included in each group, but note a too large value may cause the overflow of GPU memory.
	arguments passed through (unused)

# Value

A list of class "gwrm":

GW.arguments	a list class object including the model fitting parameters for generating the report file
GW.diagnostic	a list class object including the diagnostic information of the model fitting
lm	an object of class inheriting from "lm", see lm.
SDF	a SpatialPointsDataFrame (may be gridded), or SpatialPolygonsDataFrame object (see package "sp"), or sf object (see package "sf") integrated with regression.points, GWR coefficient estimates, y value,predicted values, coefficient standard errors and t-values in its "data" slot.
timings	starting and ending time.
this.call	the function call used.
Ftest.res	results of Leung's F tests when F123.test is TRUE.

## Note

Requirements of using CUDA for high-performence computation in GWR functions:

To run GWR-CUDA (i.e. parallel.method is pecified as "cuda") with gwr.basic , bw.gwr and gwr.model.selection, the following conditions are required:

1. There is at least one NVIDIA GPU supporting CUDA equipped on user's computer.

2. CUDA (>10.2) are installed with the environment variable 'CUDA\_HOME' set properly.

3. The package should re-built from source. - For Linux user, 'GWmodelCUDA' will be automatically built if CUDA toolkit could be detected by the complier. - For Windows user, 'GWmodel-CUDA.dll' and 'GWmodelCUDA.lib' will be automatically downloaded; however, we would recommend users to build the 'GWmodelCUDA' library manually to avoid some potentially unknown issues, and an 'CMakeLists.txt' file is provided for this procedure.

If any condition above is not satisfied, the GWR-CUDA will not work even though the "parallel.method" is specified as "cuda".

## Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

#### References

Brunsdon, C, Fotheringham, S, Charlton, M (1996), Geographically Weighted Regression: A Method for Exploring Spatial Nonstationarity. Geographical Analysis 28(4):281-298

Charlton, M, Fotheringham, S, and Brunsdon, C (2007), GWR3.0, http://gwr.nuim.ie/.

Fotheringham S, Brunsdon, C, and Charlton, M (2002), Geographically Weighted Regression: The Analysis of Spatially Varying Relationships, Chichester: Wiley.

Leung, Y, Mei, CL, and Zhang, WX (2000), Statistical tests for spatial nonstationarity based on the geographically weighted regression model. Environment and Planning A, 32, 9-32.

Lu, B, Charlton, M, Harris, P, Fotheringham, AS (2014) Geographically weighted regression with a non-Euclidean distance metric: a case study using hedonic house price data. International Journal of Geographical Information Science 28(4): 660-681

OpenMP: https://www.openmp.org/

CUDA: https://developer.nvidia.com/cuda-zone

R Core Team (2020). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. https://www.R-project.org/.

### Examples

```
data(LondonHP)
DM<-gw.dist(dp.locat=coordinates(londonhp))
##Compare the time consumed with and without a specified distance matrix
## Not run:
system.time(gwr.res<-gwr.basic(PURCHASE~FLOORSZ, data=londonhp, bw=1000,
            kernel = "gaussian"))
system.time(DM<-gw.dist(dp.locat=coordinates(londonhp)))</pre>
system.time(gwr.res<-gwr.basic(PURCHASE~FLOORSZ, data=londonhp, bw=1000,</pre>
            kernel = "gaussian", dMat=DM))
## specify an optimum bandwidth by cross-validation appraoch
bw1<-bw.gwr(PURCHASE~FLOORSZ, data=londonhp, kernel = "gaussian",dMat=DM)
gwr.res1<-gwr.basic(PURCHASE~FLOORSZ, data=londonhp, bw=bw1,kernel = "gaussian",
                   dMat=DM)
gwr.res1
## End(Not run)
data(LondonBorough)
```

```
nsa = list("SpatialPolygonsRescale", layout.north.arrow(), offset = c(561900,200900),
scale = 500, col=1)
## Not run:
if(require("RColorBrewer"))
 mypalette<-brewer.pal(6,"Spectral")</pre>
 x11()
 spplot(gwr.res1$SDF, "FLOORSZ", key.space = "right", cex=1.5, cuts=10,
 ylim=c(155840.8,200933.9), xlim=c(503568.2,561957.5),
 main="GWR estimated coefficients for FLOORSZ with a fixed bandwidth",
 col.regions=mypalette, sp.layout=list(nsa, londonborough))}
## End(Not run)
## Not run:
bw2<-bw.gwr(PURCHASE~FLOORSZ,approach="aic",adaptive=TRUE, data=londonhp,</pre>
            kernel = "gaussian", dMat=DM)
gwr.res2<-gwr.basic(PURCHASE~FLOORSZ, data=londonhp, bw=bw2,adaptive=TRUE,
                     kernel = "gaussian", dMat=DM)
gwr.res2
if(require("RColorBrewer"))
{
 x11()
 spplot(gwr.res2$SDF, "FLOORSZ", key.space = "right", cex=1.5, cuts=10,
 ylim=c(155840.8,200933.9), xlim=c(503568.2,561957.5),
 main="GWR estimated coefficients for FLOORSZ with an adaptive bandwidth",
 col.regions=mypalette, sp.layout=list(nsa,londonborough))}
## End(Not run)
## Not run:
 ############HP-GWR test code
 simulate.data.generator <- function(data.length) {</pre>
 x1 <- rnorm(data.length)</pre>
 x2 <- rnorm(data.length)</pre>
 x3 <- rnorm(data.length)</pre>
 lon <- rnorm(data.length, mean = 533200, sd = 10000)</pre>
 lat <- rnorm(data.length, mean = 159400, sd = 10000)</pre>
 y <- x1 + 5 * x2 + 2.5 * x3 + rnorm(data.length)</pre>
 simulate.data <- data.frame(y = y, x1 = x1, x2 = x2, x3 = x3, lon = lon, lat = lat)
 coordinates(simulate.data) <- ~ lon + lat</pre>
 names(simulate.data)
 return(simulate.data)
}
simulate.data <- simulate.data.generator(10000)</pre>
adaptive = TRUE
## GWR (not parallelized)
bw.CV.s <- bw.gwr(data = simulate.data, formula = y ~ x1 + x2 + x3, approach="CV",
                  kernel = "gaussian", adaptive = adaptive, parallel.method = FALSE)
model.s <- gwr.model.selection(DeVar = "y", InDeVars = c("x1", "x2", "x3"), data = simulate.data,</pre>
                          bw = bw.CV.s, approach="AIC", kernel = "gaussian", adaptive = T,
                               parallel.method = FALSE)
system.time(
```

```
betas.s <- gwr.basic(data = simulate.data, formula = y \sim x1 + x2 + x3, bw = bw.CV.s,
                       kernel = "gaussian", adaptive = TRUE)
)
## GWR-Omp
bw.CV.omp <- bw.gwr(data = simulate.data, formula = y ~ x1 + x2 + x3, approach="CV",
                    kernel = "gaussian", adaptive = adaptive, parallel.method = "omp")
model.omp <- gwr.model.selection(DeVar = "y", InDeVars = c("x1", "x2", "x3"), data = simulate.data,</pre>
                         bw = bw.CV.omp, approach="AIC", kernel = "gaussian", adaptive = T,
                                 parallel.method = "omp")
system.time(
 betas.omp <- gwr.basic(data = simulate.data, formula = y ~ x1 + x2 + x3, bw = bw.CV.omp,
                         kernel = "gaussian", adaptive = T, parallel.method = "omp"))
## GWR-CUDA
bw.CV.cuda <- bw.gwr(data = simulate.data, formula = y ~ x1 + x2 + x3, approach="CV",</pre>
                     kernel = "gaussian", adaptive = adaptive, parallel.method = "cuda",
                     parallel.arg = 6*16)
model.cuda <- gwr.model.selection(DeVar = "y", InDeVars = c("x1", "x2", "x3"),</pre>
                                  data = simulate.data, bw = bw.CV.cuda, approach="AIC",
                                  kernel = "gaussian", adaptive = T,
                                  parallel.method = "cuda", parallel.arg = 6*16)
system.time(
 betas.cuda <- gwr.basic(data = simulate.data, formula = y ~ x1 + x2 + x3, bw = bw.CV.cuda,</pre>
                           kernel = "gaussian", adaptive = T, parallel.method = "cuda",
                           parallel.arg = 6*8))
## End(Not run)
```

gwr.bootstrap Bootstrap GWR

#### Description

This function implements bootstrap methods to test for coefficient variability found from GWR under model assumptions for each of four null hypotheses: MLR, ERR, SMA and LAG models. Global test statistic results are found, as well local observation-specific test results that can be mapped.

#### Usage

# gwr.bootstrap

# Arguments

5	
formula	Regression model formula of a formula object
data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>
kernel	function chosen as follows:
	gaussian: $wgt = exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = $(1-(vdist/bw)^2)^2$ if vdist < bw, wgt=0 otherwise;
	tricube: $wgt = (1-(vdist/bw)^3)^3$ if vdist < bw, $wgt=0$ otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
approach	specified by CV for cross-validation approach or by AIC corrected (AICc) approach
R	number of random samples reapted in the bootstrap procedure
k.nearneigh	number of nearest neighbours concerned in calbrating ERR, SMA and LAG models
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist
verbose	if TRUE and bandwidth selection is undertaken, the bandwidth searches are reported
x	an object of class "gwrbsm", returned by the function gwr.bootstrap
parallel.method	1
	FALSE as default, and the calibration will be conducted traditionally via the se- rial technique, "omp": multi-thread technique with the OpenMP API, "cluster": multi-process technique with the <b>parallel</b> package, "cuda": parallel computing technique with CUDA
parallel.arg	if parallel.method is not FALSE, then set the argument by following: if paral- lel.method is "omp", parallel.arg refers to the number of threads used, and its default value is the number of cores - 1; if parallel.method is "cluster", par- allel.arg refers to the number of R sessions used, and its default value is the number of cores - 1; if parallel.method is "cuda", parallel.arg refers to the num- ber of calibrations included in each group, but note a too large value may cause the overflow of GPU memory.
	arguments passed through (unused)

# Value

A list of class "gwrbsm":

formula	Regression model formula of a formula object
results	modified statistics reported from comparisons between GWR and MLR, ERR, SMA and LAG
SDF	a SpatialPointsDataFrame (may be gridded), or SpatialPolygonsDataFrame object (see package "sp"), or sf object (see package "sf") integrated with regression.points, GWR coefficient estimates, y value,predicted values, coefficient standard errors and bootstrap p-values in its "data" slot.
timings	starting and ending time.
this.call	the function call used.

#### Note

This function implements the bootstrap methods introduced in Harris et al. (2017). It provides a global test statistic (the modified one given in Harris et al. 2017) and a complementary localised version that can be mapped. The bootstrap methods test for coefficient variability found from GWR under model assumptions for each of four null hypotheses: i) multiple linear regression model (MLR); ii) simultaneous autoregressive error model (ERR); iii) moving average error model (SMA) and iv) simultaneous autoregressive lag model (LAG).

#### Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

#### References

Harris, P., Brunsdon, C., Lu, B., Nakaya, T., & Charlton, M. (2017). Introducing bootstrap methods to investigate coefficient non-stationarity in spatial regression models. Spatial Statistics, 21, 241-261.

### Examples

```
## Not run:
#Example with the Georgia educational attainment data
data(Georgia)
data(GeorgiaCounties)
coords <- cbind(Gedu.df$X, Gedu.df$Y)</pre>
Gedu.spdf <- SpatialPointsDataFrame(coords, Gedu.df)</pre>
#Make a SpatialPolygonDataFrame
require(RColorBrewer)
gSRDF <- SpatialPolygonsDataFrame(polygons(Gedu.counties), over(Gedu.counties,
                                   Gedu.spdf),match.ID=T)
mypalette.1 <- brewer.pal(11,"Spectral")</pre>
X11(width=9,height=8)
spplot(gSRDF, names(gSRDF)[c(5,7:9)], col.regions=mypalette.1,
cuts=10, par.settings=list(fontsize=list(text=15)),
main=expression(paste("Georgia educational attainment predictor data")))
bsm.res <- gwr.bootstrap(PctBach~PctRural+PctEld+PctFB+PctPov, gSRDF,</pre>
                         R=999, longlat=T)
bsm.res
#local bootstrap tests with respect to: MLR, ERR, SMA and LAG models.
```

```
mypalette.local.test <- brewer.pal(10,"Spectral")</pre>
X11(width=12,height=16)
spplot(bsm.res$SDF, names(bsm.res$SDF)[14:17], col.regions=mypalette.local.test,
cuts=9, par.settings=list(fontsize=list(text=15)),
main=expression(paste("Local p-values for each coefficient of the MLR model
                      null hypothesis")))
X11(width=12,height=16)
spplot(bsm.res$SDF, names(bsm.res$SDF)[19:22], col.regions=mypalette.local.test,
cuts=9, par.settings=list(fontsize=list(text=15)),
main=expression(paste("Local p-values for each coefficient of the ERR model
                      null hypothesis")))
X11(width=12,height=16)
spplot(bsm.res$SDF, names(bsm.res$SDF)[24:27], col.regions=mypalette.local.test,
cuts=9, par.settings=list(fontsize=list(text=15)),
main=expression(paste("Local p-values for each coefficient of the SMA model null
                      hypothesis")))
X11(width=12,height=16)
spplot(bsm.res$SDF, names(bsm.res$SDF)[29:32], col.regions=mypalette.local.test,
cuts=9, par.settings=list(fontsize=list(text=15)),
main=expression(paste("Local p-values for each coefficient of the LAG model null
                      hypothesis")))
#Example with Dublin voter data
data(DubVoter)
X11(width=9,height=8)
spplot(Dub.voter, names(Dub.voter)[c(5,7,9,10)], col.regions=mypalette.1,
cuts=10, par.settings=list(fontsize=list(text=15)),
main=expression(paste("Dublin voter turnout predictor data")))
bsm.res1 <- gwr.bootstrap(GenEl2004~LARent+Unempl+Age18_24+Age25_44, Dub.voter</pre>
                        , R=999)
bsm.res1
#local bootstrap tests with respect to: MLR, ERR, SMA and LAG models.
X11(width=11,height=8)
spplot(bsm.res1$SDF, names(bsm.res1$SDF)[14:17], col.regions=mypalette.local.test,
cuts=9, par.settings=list(fontsize=list(text=15)),
main=expression(paste("Local p-values for each coefficient of the MLR model null
                       hypothesis")))
X11(width=11,height=8)
spplot(bsm.res1$SDF, names(bsm.res1$SDF)[19:22], col.regions=mypalette.local.test,
cuts=9, par.settings=list(fontsize=list(text=15)),
main=expression(paste("Local p-values for each coefficient of the ERR model null
                       hypothesis")))
X11(width=11,height=8)
spplot(bsm.res1$SDF, names(bsm.res1$SDF)[24:27], col.regions=mypalette.local.test,
cuts=9, par.settings=list(fontsize=list(text=15)),
main=expression(paste("Local p-values for each coefficient of the SMA model
                           null hypothesis")))
X11(width=11,height=8)
spplot(bsm.res1$SDF, names(bsm.res1$SDF)[29:32], col.regions=mypalette.local.test,
cuts=9, par.settings=list(fontsize=list(text=15)),
```

## End(Not run)

gwr.collin.diagno Local collinearity diagnostics for basic GWR

## Description

This function provides a series of local collinearity diagnostics for the independent variables of a basic GWR model.

## Usage

## Arguments

formula	Regression model formula of a formula object
data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>
bw	bandwidth used in the weighting function, probably calculated by bw.gwr or bw.gwr.lcr; fixed (distance) or adaptive bandwidth (number of nearest neighbours)
kernel	function chosen as follows:
	gaussian: wgt = $exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
	tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist

48

### gwr.cv

#### Value

corr.mat	Local correlation matrix
VIF	Local Variance inflation factors (VIFs) matrix
local_CN	Local condition numbers
VDP	Local variance-decomposition proportions
SDF	a SpatialPointsDataFrame (may be gridded), or SpatialPolygonsDataFrame object (see package "sp"), or sf object (see package "sf") integrated with VIF, local_CN, VDP and corr.mat

# Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

## References

Wheeler D, Tiefelsdorf M (2005) Multicollinearity and correlation among local regression coefficients in geographically weighted regression. Journal of Geographical Systems 7:161-187

Wheeler D (2007) Diagnostic tools and a remedial method for collinearity in geographically weighted regression. Environment and Planning A 39:2464-2481

Gollini I, Lu B, Charlton M, Brunsdon C, Harris P (2015) GWmodel: an R Package for exploring Spatial Heterogeneity using Geographically Weighted Models. Journal of Statistical Software, 63(17):1-50

gwr.cv

Cross-validation score for a specified bandwidth for basic GWR

## Description

This function finds the cross-validation score for a specified bandwidth for basic GWR. It can be used to construct the bandwidth function across all possible bandwidths and compared to that found automatically.

#### Usage

bw	bandwidth used in the weighting function; fixed (distance) or adaptive band-width(number of nearest neighbours)
Х	a numeric matrix of the independent data with an extra column of "ones" for the 1st column
Υ	a column vector of the dependent data

kernel	<pre>function chosen as follows: gaussian: wgt = exp(5*(vdist/bw)^2); exponential: wgt = exp(-vdist/bw); bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist &lt; bw, wgt=0 otherwise;</pre>
	tricube: wgt = $(1-(vdist/bw)^3)^3$ if vdist < bw, wgt=0 otherwise; boxcar: wgt=1 if dist < bw, wgt=0 otherwise
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
dp.locat	a two-column numeric array of observation coordinates
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist
verbose	if TRUE (default), reports the progress of search for bandwidth
parallel.method	
	Specified by 'FALSE' for serial approach, by '"omp"' for multi-thread approach implemented via OpenMP, by '"cluster"' for multi-process approach implemented via 'parallel' package, by '"cuda"' for parallel approach implemented via CUDA
parallel.arg	Set the argument for parallel approach. If 'parallel.method' is 'FALSE', there is no need to set its value. If 'parallel.method' is '"omp"', its value is used to set how many threads should be created (default by cores of *cores of CPU* - 1). If 'parallel.method' is '"cluster"', its value is used to set how many R session should be created (default by cores of *cores of CPU* - 1). If 'parallel.method' is '"cuda"', its value is used to set how many samples is included in one group during the calibration. This value should not be too big to avoid the overflow of GPU memory.
Value	
CV.score	cross-validation score
Author(s)	
Binbin Lu <binbi< td=""><td>nlu@whu.edu.cn&gt;</td></binbi<>	nlu@whu.edu.cn>

gwr.cv.contrib	Cross-validation data at each observation location for a basic GWR
	model

# Description

This function finds the individual cross-validation score at each observation location, for a basic GWR model, for a specified bandwidth. These data can be mapped to detect unusually high or low cross-validations scores.

## gwr.cv.contrib

# Usage

# Arguments

bw	bandwidth used in the weighting function; fixed (distance) or adaptive band-width(number of nearest neighbours)	
Х	a numeric matrix of the independent data with an extra column of "ones" for the 1st column	
Υ	a column vector of the dependent data	
kernel	function chosen as follows:	
	gaussian: wgt = exp(5*(vdist/bw)^2);	
	exponential: wgt = exp(-vdist/bw);	
	bisquare: wgt = $(1-(vdist/bw)^2)^2$ if vdist < bw, wgt=0 otherwise;	
	tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;	
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise	
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)	
dp.locat	a two-column numeric array of observation coordinates	
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance	
theta	an angle in radians to rotate the coordinate system, default is 0	
longlat	if TRUE, great circle distances will be calculated	
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist	
parallel.method		
	Specified by 'FALSE' for serial approach, by '"omp"' for multi-thread approach implemented via OpenMP, by '"cluster"' for multi-process approach implemented via 'parallel' package, by '"cuda"' for parallel approach implemented via CUDA	
parallel.arg	Set the argument for parallel approach. If 'parallel.method' is 'FALSE', there is no need to set its value. If 'parallel.method' is '"omp"', its value is used to set how many threads should be created (default by cores of *cores of CPU* - 1). If 'parallel.method' is '"cluster"', its value is used to set how many R session should be created (default by cores of *cores of CPU* - 1). If 'parallel.method' is '"cuda"', its value is used to set how many samples is included in one group during the calibration. This value should not be too big to avoid the overflow of GPU memory.	

## Value

CV a data vector consisting of squared residuals, whose sum is the cross-validation score for the specified bandwidth.

## Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

gwr.hetero Heteroskedastic GWR

# Description

This function implements a heteroskedastic GWR model

## Usage

formula	Regression model formula of a formula object
data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>
regression.poi	nts
	a Spatial*DataFrame object, i.e. SpatialPointsDataFrame or SpatialPolygons- DataFrame as defined in package <b>sp</b>
bw	bandwidth used in the weighting function, possibly calculated by bw.gwr;fixed (distance) or adaptive bandwidth(number of nearest neighbours)
kernel	function chosen as follows:
	gaussian: wgt = $exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
	tricube: $wgt = (1-(vdist/bw)^3)^3$ if vdist < bw, $wgt=0$ otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
tol	the threshold that determines the convergence of the iterative procedure
maxiter	the maximum number of times to try the iterative procedure
verbose	logical, if TRUE verbose output will be made from the iterative procedure
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist

#### gwr.lcr

#### Value

SDF

a SpatialPointsDataFrame (may be gridded), or SpatialPolygonsDataFrame object (see package "sp"), or sf object (see package "sf") integrated with coefficient estimates in its "data" slot.

#### Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

## References

Fotheringham S, Brunsdon, C, and Charlton, M (2002), Geographically Weighted Regression: The Analysis of Spatially Varying Relationships, Chichester: Wiley.

Harris P, Fotheringham AS, Juggins S (2010) Robust geographically weighed regression: a technique for quantifying spatial relationships between freshwater acidification critical loads and catchment attributes. Annals of the Association of American Geographers 100(2): 286-306

Harris P, Brunsdon C, Fotheringham AS (2011) Links, comparisons and extensions of the geographically weighted regression model when used as a spatial predictor. Stochastic Environmental Research and Risk Assessment 25:123-138

gwr.lcr

GWR with a locally-compensated ridge term

#### Description

To address possible local collinearity problems in basic GWR, GWR-LCR finds local ridge parameters at affected locations (set by a user-specified threshold for the design matrix condition number).

### Usage

formula	Regression model formula of a formula object	
data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>	
regression.points		
	a Spatial*DataFrame object, i.e. SpatialPointsDataFrame or SpatialPolygons- DataFrame as defined in package <b>sp</b> , or a two-column numeric array	
bw	bandwidth used in the weighting function, possibly calculated by bw.gwr.lcr; fixed (distance) or adaptive bandwidth(number of nearest neighbours)	

kernel	function chosen as follows:
	gaussian: $wgt = exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: $wgt = (1-(vdist/bw)^2)^2$ if vdist < bw, wgt=0 otherwise;
	tricube: $wgt = (1-(vdist/bw)^3)^3$ if vdist < bw, $wgt=0$ otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
lambda	option for a globally-defined (constant) ridge parameter. Default is lambda=0, which gives a basic GWR fit
lambda.adjust	a locally-varying ridge parameter. Default FALSE, refers to: (i) a basic GWR without a local ridge adjustment (i.e. lambda=0, everywhere); or (ii) a penalised GWR with a global ridge adjustment (i.e. lambda is user-specified as some constant, other than 0 everywhere); if TRUE, use cn.tresh to set the maximum condition number. Here for locations with a condition number (for its local design matrix) above this user-specified threshold, a local ridge parameter is found
cn.thresh	maximum value for condition number, commonly set between 20 and 30
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
cv	if TRUE, 'cross-validation data will be calculated and returned in the output Spatial*DataFrame
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist
x	an object of class "gwrlcr", returned by the function gwr.lcr
	arguments passed through (unused)

# Value

A list of class "rgwr":

SDF	a SpatialPointsDataFrame (may be gridded), or SpatialPolygonsDataFrame object (see package "sp"), or sf object (see package "sf") with coordinates of regression.points in its "data" slot.
GW.arguments	parameters used for the LCR-GWR calibration
GW.diagnostic	diagnostic information is given when data points are also used as regression locations
timings	timing information for running this function
this.call	the function call used.

# Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

gwr.lcr

### References

Wheeler D (2007) Diagnostic tools and a remedial method for collinearity in geographically weighted regression. Environment and Planning A 39:2464-2481

Brunsdon C, Charlton M, Harris P (2012) Living with collinearity in Local Regression Models. GISRUK 2012, Lancaster, UK

Brunsdon C, Charlton M, Harris P (2012) Living with collinearity in Local Regression Models. Spatial Accuracy 2012, Brazil

Gollini I, Lu B, Charlton M, Brunsdon C, Harris P (2015) GWmodel: an R Package for exploring Spatial Heterogeneity using Geographically Weighted Models. Journal of Statistical Software 63(17): 1-50

### Examples

```
data(DubVoter)
require(RColorBrewer)
# Function to find the global condition number (CN)
BKW_cn <- function (X) {
  p <- dim(X)[2]
  Xscale <- sweep(X, 2, sqrt(colSums(X^2)), "/")</pre>
  Xsvd <- svd(Xscale)$d</pre>
  cn <- Xsvd[1] / Xsvd[p]</pre>
  cn
}
#
X <- cbind(1,Dub.voter@data[,3:10])</pre>
head(X)
CN.global <- BKW_cn(X)
CN.global
## Not run:
# gwr.lcr function with a global bandwidth to check that the global CN is found
gwr.lcr1 <- gwr.lcr(GenEl2004~DiffAdd+LARent+SC1+Unempl+LowEduc+Age18_24</pre>
+Age25_44+Age45_64, data=Dub.voter, bw=1000000000)
summary(gwr.lcr1$SDF$Local_CN)
# Find and map the local CNs from a basic GWR fit using the lcr-gwr function
#(note this is NOT the locally-compensated ridge GWR fit as would need to set
#lambda.adjust=TRUE and cn.thresh=30, say)
bw.lcr2 <- bw.gwr.lcr(GenEl2004~DiffAdd+LARent+SC1+Unempl+LowEduc+Age18_24</pre>
+Age25_44+Age45_64, data=Dub.voter, kernel="bisquare", adaptive=TRUE)
gwr.lcr2 <- gwr.lcr(GenEl2004~DiffAdd+LARent+SC1+Unempl+LowEduc+Age18_24</pre>
+Age25_44+Age45_64, data=Dub.voter, bw=bw.lcr2, kernel="bisquare", adaptive=TRUE)
if(require("RColorBrewer"))
  spplot(gwr.lcr2$SDF,"Local_CN",col.regions=brewer.pal(9,"YlOrRd"),cuts=8,
  main="Local CN")
```

## End(Not run)

gwr.lcr.cv

## Description

This function finds the cross-validation score for a specified bandwidth for GWR-LCR. It can be used to construct the bandwidth function across all possible bandwidths and compared to that found automatically.

# Usage

bw	bandwidth used in the weighting function; fixed (distance) or adaptive band-width(number of nearest neighbours)
Х	a numeric matrix of the independent data with an extra column of "ones" for the 1st column
Υ	a column vector of the dependent data
kernel	function chosen as follows:
	gaussian: $wgt = exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
	tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
locs	a two-column numeric array of observation coordinates
lambda	option for a globally-defined (constant) ridge parameter. Default is lambda=0, which gives a basic GWR fit
lambda.adjust	a locally-varying ridge parameter. Default FALSE, refers to: (i) a basic GWR without a local ridge adjustment (i.e. lambda=0, everywhere); or (ii) a penalised GWR with a global ridge adjustment (i.e. lambda is user-specified as some constant, other than 0 everywhere); if TRUE, use cn.tresh to set the maximum condition number. Here for locations with a condition number (for its local design matrix) above this user-specified threshold, a local ridge parameter is found
cn.thresh	maximum value for condition number, commonly set between 20 and 30
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance

## gwr.lcr.cv.contrib

theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function $gw.dist$

## Value

CV.score cross-validation score

## Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

gwr.lcr.cv.contrib	Cross-validation data at each observation location for the GWR-LCR
	model

# Description

This function finds the individual cross-validation score at each observation location, for a GWR-LCR model, for a specified bandwidth. These data can be mapped to detect unusually high or low cross-validations scores.

## Usage

bw	bandwidth used in the weighting function; fixed (distance) or adaptive band- width(number of nearest neighbours)
Х	a numeric matrix of the independent data with an extra column of "ones" for the 1st column
Υ	a column vector of the dependent data
locs	a two-column numeric array of observation coordinates
kernel	function chosen as follows:
	gaussian: wgt = $exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
	tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
lambda	option for a globally-defined (constant) ridge parameter. Default is lambda=0, which gives a basic GWR fit

lambda.adjust	a locally-varying ridge parameter. Default FALSE, refers to: (i) a basic GWR without a local ridge adjustment (i.e. lambda=0, everywhere); or (ii) a penalised GWR with a global ridge adjustment (i.e. lambda is user-specified as some constant, other than 0 everywhere); if TRUE, use cn.tresh to set the maximum condition number. Here for locations with a condition number (for its local design matrix) above this user-specified threshold, a local ridge parameter is found
cn.thresh	maximum value for condition number, commonly set between 20 and 30
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function $gw.dist$

## Value

CV	a data vector consisting of squared residuals, whose sum is the cross-validation
	score for the specified bandwidth.

### Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

gwr.mink.approach Minkovski approach for GWR

## Description

This function implements the Minkovski approach to select an 'optimum' distance metric for calibrating a GWR model.

## Usage

# Arguments

-	
formula	Regression model formula of a formula object
data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>
criterion	the criterion used for distance metric selection, AICc ("AICc") or cross-validation ("CV") score; default is "AICc"
bw	bandwidth used in the weighting function, possibly calculated by bw.gwr;fixed (distance) or adaptive bandwidth(number of nearest neighbours)
bw.sel.approac	h
	approach used to seclect an optimum bandwidth for each calibration if no band- width (bw) is given; specified by CV for cross-validation approach or by AIC corrected (AICc) approach
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
kernel	function chosen as follows:
	gaussian: wgt = $exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
	tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
p.vals	a collection of positive numbers used as the power of the Minkowski distance
p.inf	if TRUE, Chebyshev distance is tried for model calibration, i.e. p is infinity
theta.vals	a collection of values used as angles in radians to rotate the coordinate system
verbose	if TRUE and bandwidth selection is undertaken, the bandwidth searches are reported
nlower	the minmum number of nearest neighbours if an adaptive kernel is used

## Value

A list of:	
diag.df	a data frame with four columns (p, theta, bandwidth, AICc/CV), each row corresponds to a calibration
coefs.all	a list class object including all the estimated coefficients

# Note

The function "mink.approach" (in the early versions of GWmodel) has been renamed as "gwr.mink.approach", while the old name is still kept valid.

# Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

#### References

Lu, B, Charlton, M, Brunsdon, C & Harris, P(2016). The Minkowski approach for choosing the distance metric in Geographically Weighted Regression. International Journal of Geographical Information Science, 30(2): 351-368.

gwr.mink.matrixview Visualisation of the results from gwr.mink.approach

## Description

This function visualises the AICc/CV results from the gwr.mink.approach.

#### Usage

```
gwr.mink.matrixview(diag.df, znm=colnames(diag.df)[4], criterion="AIC")
```

#### Arguments

diag.df	the first part of a list object returned by gwr.mink.approach
znm	the name of the forth column in diag.df
criterion	the criterion used for distance metric selection in gwr.mink.approach

#### Note

The function "mink.matrixview" (in the early versions of GWmodel) has been renamed as "gwr.mink.matrixview", while the old name is still kept valid.

## Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

### References

Lu, B, Charlton, M, Brunsdon, C & Harris, P(2016). The Minkowski approach for choosing the distance metric in Geographically Weighted Regression. International Journal of Geographical Information Science, 30(2): 351-368.

gwr.mink.pval

#### Description

These functions implement heuristics to select the values of p from two intervals: (0, 2] in a 'back-ward' direction and (2, Inf) in a 'forward' direction.

#### Usage

formula	Regression model formula of a formula object
data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b>
criterion	the criterion used for distance metric selection, AICc ("AICc") or cross-validation ("CV") score; default is "AICc"
bw	bandwidth used in the weighting function, possibly calculated by bw.gwr;fixed (distance) or adaptive bandwidth(number of nearest neighbours)
bw.sel.approac	h
	approach used to seclect an optimum bandwidth for each calibration if no band- width (bw) is given; specified by CV for cross-validation approach or by AIC corrected (AICc) approach
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
kernel	<pre>function chosen as follows: gaussian: wgt = exp(5*(vdist/bw)^2); exponential: wgt = exp(-vdist/bw); bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist &lt; bw, wgt=0 otherwise; tricube: wgt = (1-(vdist/bw)^3)^3 if vdist &lt; bw, wgt=0 otherwise; boxcar: wgt=1 if dist &lt; bw, wgt=0 otherwise</pre>
	bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;

left.interval	the step-size for searching the left interval (0, 2] in a 'backward' direction
right.interval	the step-size for searching the right interval (2, Inf) in a 'forward' direction
p.max	the maximum value of p
p.min	the minimum value of p
interval	the step-size for searching the given interval in a 'backward' or 'forward' direc- tion
drop.tol	an AICc difference threshold to define whether the values of p to be dropped or not
theta0	a fixed rotation angle in radians
verbose	if TRUE and bandwidth selection is undertaken, the bandwidth searches are reported
nlower	the minmum number of nearest neighbours if an adaptive kernel is used
х	an object of class "pvlas", returned by these functions
	arguments passed through (unused)

### Value

A list of:	
p.vals	a vector of tried values of p
cretion.vals	a vector of criterion values (AICc or CV) for tried values of p
p.dropped	a vector of boolean to label whether a value of p to be dropped or not: TRUE means to be dropped and FALSE means to be used for the Minkowski approach

## Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

## References

Lu, B, Charlton, M, Brunsdon, C & Harris, P(2016). The Minkowski approach for choosing the distance metric in Geographically Weighted Regression. International Journal of Geographical Information Science, 30(2): 351-368.

gwr.mixed

Mixed GWR

## Description

This function implements mixed (semiparametric) GWR

## Usage

# gwr.mixed

# Arguments

formula	Regression model formula of a formula object
data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>
regression.poi	nts
	a Spatial*DataFrame object, i.e. SpatialPointsDataFrame or SpatialPolygons- DataFrame as defined in package <b>sp</b>
fixed.vars	independent variables that appeared in the formula that are to be treated as global
intercept.fixe	d
	logical, if TRUE the intercept will be treated as global
bw	bandwidth used in the weighting function, possibly calculated by bw.gwr;fixed (distance) or adaptive bandwidth(number of nearest neighbours)
diagnostic	logical, if TRUE the diagnostics will be calculated
kernel	function chosen as follows:
	gaussian: wgt = $exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
	tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist
dMat.rp	a distance matrix when an individual set of regression points are adopted

# Value

A list of class "mgwr":

GW.arguments	a list class object including the model fitting parameters for generating the report file
aic	AICc value from this calibration
df.used	effective degree of freedom
rss	residual sum of squares
SDF	a SpatialPointsDataFrame (may be gridded), or SpatialPolygonsDataFrame object (see package "sp"), or sf object (see package "sf") integrated with coefficient estimates in its "data" slot.
timings	starting and ending time.
this.call	the function call used.

For an alternative formulation of mixed GWR, please refer to GWR 4, which provides useful tools for automatic bandwidth selection. This windows-based software also implements generalised mixed GWR.

The mixed GWR in the latest release of GWmodel (2.0-0) has been revised by Dr. Fiona H Evans from Centre for Digital Agriculture, Murdoch and Curtin Universities in terms of its computational efficiency.

#### Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

#### References

Fotheringham S, Brunsdon, C, and Charlton, M (2002), Geographically Weighted Regression: The Analysis of Spatially Varying Relationships, Chichester: Wiley.

Brunsdon C, Fotheringham AS, Charlton ME (1999) Some notes on parametric significance tests for geographically weighted regression. Journal of Regional Science 39(3):497-524

Mei L-M, He S-Y, Fang K-T (2004) A note on the mixed geographically weighted regression model. Journal of regional science 44(1):143-157

Mei L-M, Wang N, Zhang W-X (2006) Testing the importance of the explanatory variables in a mixed geographically weighted regression model. Environment and Planning A 38:587-598

Nakaya T, Fotheringham AS, Brunsdon C, Charlton M (2005) Geographically Weighted Poisson Regression for Disease Association Mapping, Statistics in Medicine 24: 2695-2717

Nakaya T et al. (2011) GWR4.0, http://gwr.nuim.ie/.

gwr.model.selection Model selection for GWR with a given set of independent variables

## Description

This function selects one GWR model from many alternatives based on the AICc values.

#### Usage

64

## Note

# Arguments

DeVar	dependent variable
InDeVars	a vector of independent variables for model selection
data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package $\mathbf{sp}$ , or a sf object defined in package $\mathbf{sf}$
bw	bandwidth used in the weighting function, possibly calculated by bw.gwr
approach	specified by CV (cv) for cross validation approach or AIC (aic) for selecting bandwidth by AICc values
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
kernel	function chosen as follows:
	gaussian: wgt = $exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = $(1-(vdist/bw)^2)^2$ if vdist < bw, wgt=0 otherwise;
	tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
parallel.method	
	Specified by 'FALSE' for serial approach, by '"omp"' for multi-thread approach implemented via OpenMP, by '"cluster"' for multi-process approach implemented via 'parallel' package, by '"cuda"' for parallel approach implemented via CUDA
parallel.arg	Set the argument for parallel approach. If 'parallel.method' is 'FALSE', there is no need to set its value. If 'parallel.method' is '"omp"', its value is used to set how many threads should be created (default by cores of *cores of CPU* - 1). If 'parallel.method' is '"cluster"', its value is used to set how many R session should be created (default by cores of *cores of CPU* - 1). If 'parallel.method' is '"cuda"', its value is used to set how many samples is included in one group during the calibration. This value should not be too big to avoid the overflow of GPU memory.
10	

# Value

A list of:

model.list	a list of all the tried GWR models consisted of formulas and variables.
GWR.df	a data frame consited of four columns: bandwidth, AIC, AICc, RSS

#### Note

The algorithm for selecting GWR models consists of the following four steps:

Step 1. Start by calibrating all the possible bivariate GWR models by sequentially regressing a single independent variable against the dependent variable;

Step 2. Find the best performing model which produces the minimum AICc value, and permanently include the corresponding independent variable in subsequent models;

Step 3. Sequentially introduce a variable from the remaining group of independent variables to construct new models with the permanently included independent variables, and determine the next permanently included variable from the best fitting model that has the minimum AICc value;

Step 4. Repeat step 3 until all the independent variables are permanently included in the model.

In this procedure, the independent variables are iteratively included into the model in a "forward" direction. Note that there is a clear distinction between the different number of involved variables in a selection step, which can be called model levels.

## Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

## References

Lu, B, Charlton, M, Harris, P, Fotheringham, AS (2014) Geographically weighted regression with a non-Euclidean distance metric: a case study using hedonic house price data. International Journal of Geographical Information Science 28(4): 660-681

#### See Also

gwr.model.view, gwr.model.sort

gwr.model.sort Sort the results of the GWR model selection function gwr.model.selection.

#### Description

Sort the results from the GWR model selection function gwr.model.selection

#### Usage

gwr.model.sort(Sorting.list , numVars, ruler.vector)

Sorting.list	a list returned by function gwr.model.selection
numVars	the number of independent variables involved in model selection
ruler.vector	a numeric vector as the sorting basis

## Note

The function sorts the results of model selection within individual levels.

The function "model.sort.gwr" (in the early versions of GWmodel) has been renamed as "gwr.model.sort", while the old name is still kept valid.

#### Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

#### See Also

gwr.model.selection,gwr.model.view

gwr.model.view Visualise the GWR models from gwr.model.selection

## Description

This function visualises the GWR models from gwr.model.selection.

### Usage

gwr.model.view(DeVar, InDeVars, model.list)

## Arguments

DeVar	dependent variable
InDeVars	a vector of independent variables for model selection
model.list	a list of all GWR model tried in gwr.model.selection

## Note

The function "model.view.gwr" (in the early versions of GWmodel) has been renamed as "gwr.model.view", while the old name is still kept valid.

## Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

## See Also

gwr.model.selection,gwr.model.sort

# Examples

```
## Not run:
data(LondonHP)
DM<-gw.dist(dp.locat=coordinates(londonhp))
DeVar<-"PURCHASE"
InDeVars<-c("FLOORSZ","GARAGE1","BLDPWW1","BLDPOSTW")
model.sel<-gwr.model.selection(DeVar,InDeVars, data=londonhp,
kernel = "gaussian", dMat=DM,bw=5000)
model.list<-model.sel[[1]]
gwr.model.view(DeVar, InDeVars, model.list=model.list)
```

## End(Not run)

gwr.montecarlo	Monte Carlo (randomisation) test for significance of GWR parameter
	variability

## Description

This function implements a Monte Carlo (randomisation) test to test for significant (spatial) variability of a GWR model's parameters or coefficients.

## Usage

# Arguments

formula	Regression model formula of a formula object
data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>
nsims	the number of randomisations
kernel	function chosen as follows:
	gaussian: wgt = $exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
	tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
bw	bandwidth used in the weighting function, possibly calculated by bw.gwr
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist

68

#### gwr.multiscale

#### Value

pmat

A vector containing p-values for all the GWR parameters

## Note

The function "montecarlo.gwr" (in the early versions of GWmodel) has been renamed as "gwr.montecarlo", while the old name is still kept valid.

#### Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

#### References

Brunsdon C, Fotheringham AS, Charlton ME (1998) Geographically weighted regression - modelling spatial non-stationarity. Journal of the Royal Statistical Society, Series D-The Statistician 47(3):431-443

Fotheringham S, Brunsdon, C, and Charlton, M (2002), Geographically Weighted Regression: The Analysis of Spatially Varying Relationships, Chichester: Wiley.

Charlton, M, Fotheringham, S, and Brunsdon, C (2007), GWR3.0.

#### Examples

```
## Not run:
data(LondonHP)
DM<-gw.dist(dp.locat=coordinates(londonhp))
bw<-bw.gwr(PURCHASE~FLOORSZ,data=londonhp,dMat=DM, kernel="gaussian")
#See any difference in the next two commands and why?
res.mont1<-gwr.montecarlo(PURCHASE~PROF+FLOORSZ, data = londonhp,dMat=DM,
nsim=99, kernel="gaussian", adaptive=FALSE, bw=3000)
res.mont2<-gwr.montecarlo(PURCHASE~PROF+FLOORSZ, data = londonhp,dMat=DM,
nsim=99, kernel="gaussian", adaptive=FALSE, bw=30000000000)
```

## End(Not run)

gwr.multiscale Multiscale GWR

## Description

This function implements multiscale GWR to detect variations in regression relationships across different spatial scales. This function can not only find a different bandwidth for each relationship but also (and simultaneously) find a different distance metric for each relationship (if required to do so).

## Usage

## Arguments

formula	Regression model formula of a formula object
data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>
kernel	function chosen as follows:
	gaussian: wgt = $exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = $(1-(vdist/bw)^2)^2$ if vdist < bw, wgt=0 otherwise;
	tricube: $wgt = (1-(vdist/bw)^3)^3$ if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
criterion	criterion for determining the convergence of the back-fitting procedure, could be "CVR" or "dCVR", which corespond to the changing value of RSS (CVR) and the differential version (dCVR), respectively; and "dCVR" is used as default.
max.iterations	maximum number of iterations in the back-fitting procedure
threshold	threshold value to terminate the back-fitting iterations
dMats	a list of distance matrices used for estimating each specific parameter
var.dMat.indx	index corresponds to a specific distance matrix for each exploratory variable, if dMats is provided
p.vals	a collection of positive numbers used as the power of the Minkowski distance
theta.vals	a collection of values used as angles in radians to rotate the coordinate system
longlat	if TRUE, great circle distances will be calculated
bws0	a vector of initializing bandwidths for the back-fitting procedure, of which the length should equal to the number of paramters if specified
bw.seled	a vector of boolean variables to determine whether the corresponding bandwidth should be re-selected or not: if TRUE, the corresponding bandwiths for the specific parameters are supposed to be given in bws0; otherwise, the bandwidths for the specific parameters will be selected within the back-fitting iterations.

70

approach	specified by CV for cross-validation approach or by AIC corrected (AICc) approach
bws.thresholds	threshold values to define whether the bandwidth for a specific parameter has converged or not
bws.reOpts	the number times of continually optimizing each parameter-specific bandwidth even though it meets the criterion of convergence, for avoiding sub-optimal choice due to illusion of convergence;
verbose	if TRUE and bandwidth selection is undertaken, the bandwidth searches are reported
predictor.cente	ered
	a logical vector of length equalling to the number of predictors, and note inter- cept is not included; if the element is TRUE, the corresponding predictor will be centered.
hatmatrix	if TRUE the hatmatrix for the whole model will be calculated, and AICc, adjusted-R2 values will be returned accordingly.
nlower	the minmum number of nearest neighbours if an adaptive kernel is used
parallel.method	i
	FALSE as default, and the calibration will be conducted traditionally via the se- rial technique, "omp": multi-thread technique with the OpenMP API, "cluster": multi-process technique with the <b>parallel</b> package, "cuda": parallel computing technique with CUDA
parallel.arg	if parallel.method is not FALSE, then set the argument by following: if parallel.method is "omp", parallel.arg refers to the number of threads used, and its default value is the number of cores - 1; if parallel.method is "cluster", parallel.arg refers to the number of R sessions used, and its default value is the number of cores - 1; if parallel.method is "cuda", parallel.arg refers to the number of calibrations included in each group, but note a too large value may cause the overflow of GPU memory.
force.armadillo	)
	if TRUE, use the original RcppArmadillo implementation instead of the new RcppEigen implementation. Only matters if parallel.method = F or parallel.method = "omp".
x	an object of class "multiscalegwr", returned by the function gwr.multiscale
	arguments passed through (unused)
Value	

A list of class "psdmgwr":

SDF	a SpatialPointsDataFrame (may be gridded), or SpatialPolygonsDataFrame object (see package "sp"), or sf object (see package "sf") integrated with data locations, coefficient estimates from the PSDM GWR model, predicted y values, residuals, coefficient standard errors and t-values in its "data" slot.
GW.arguments	a list class object including the model fitting parameters for generating the report file

GW.diagnostic	a list class object including the diagnostic information of the model fitting
lm	an object of class inheriting from "lm", see lm.
bws.vars	bandwidths used for all the parameters within the back-fitting procedure
timings	starting and ending time.
this.call	the function call used.

#### Note

This function implements multiscale GWR to detect variations in regression relationships across different spatial scales. This function can not only find a different bandwidth for each relationship, but also (and simultaneously), find a different distance metric for each relationship (i.e. Parameter-Specific Distance Metric GWR, i.e. PSDM GWR). Note that multiscale GWR (MGWR) has also been referred to as flexible bandwidth GWR (FBGWR) and conditional GWR (CGWR) in the literature. All are one and the same model, but where PSDM-GWR additionally provides a different distance metric option for each relationship. An MGWR model is calibrated if no "dMats" and "p.vals" are specified; a mixed GWR model will be calibrated if an infinite bandwidth and another regular bandwidth are used for estimating the global and local parameters (again when no "dMats" and "p.vals" are specified). In other words, the gwr.multiscale function is specified with Euclidean distances in both cases. Note that the results from this function for a mixed GWR model and gwr.mixed might be different, as a back-fitting algorithm is used in gwr.multiscale, while an approximating algorithm is applied in gwr.mixed. The gwr.mixed function performs better in computational efficiency, but poorer in prediction accuracy.

## Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

#### References

Yang, W. (2014). An Extension of Geographically Weighted Regression with Flexible Bandwidths. St Andrews, St Andrews, UK.

Lu, B., Harris, P., Charlton, M., & Brunsdon, C. (2015). Calibrating a Geographically Weighted Regression Model with Parameter-specific Distance Metrics. Procedia Environmental Sciences, 26, 109-114.

Lu, B., Brunsdon, C., Charlton, M., & Harris, P. (2017). Geographically weighted regression with parameter-specific distance metrics. International Journal of Geographical Information Science, 31, 982-998.

Fotheringham, A. S., Yang, W. & Kang, W. (2017). Multiscale Geographically Weighted Regression (MGWR). Annals of the American Association of Geographers, 107, 1247-1265.

Yu, H., A. S. Fotheringham, Z. Li, T. Oshan, W. Kang & L. J. Wolf. 2019. Inference in multiscale geographically weighted regression. Geographical Analysis(In press).

Leong, Y.Y., & Yue, J.C. (2017). A modification to geographically weighted regression. International Journal of Health Geographics, 16 (1), 11.

Lu, B., Yang, W. Ge, Y. & Harris, P. (2018). Improvements to the calibration of a geographically weighted regression with parameter-specific distance metrics and bandwidths. Forthcoming Computers, Environment and Urban Systems.
Wolf, L.J, Oshan, T.M, Fotheringham, A.S. (2018). Single and multiscale models of process spatial heterogeneity. Geographical Analysis, 50(3): 223-246.

Murakami, D., Lu, B., Harris, P., Brunsdon, C., Charlton, M., Nakaya, T., & Griffith, D. (2019) The importance of scale in spatially varying coefficient modelling. Forthcoming Annals of the Association of American Geographers.

## Examples

```
data(LondonHP)
EUDM <- gw.dist(coordinates(londonhp))</pre>
#No bandwidth is selected, and bws0 values are used
## Not run:
###Similar as the basic GWR
res1<-gwr.multiscale(PURCHASE~FLOORSZ+PROF, data=londonhp, criterion="dCVR",kernel="gaussian",
adaptive=T, bws0=c(100, 100, 100),bw.seled=rep(T, 3), dMats=list(EUDM,EUDM,EUDM))
#FBGWR
res2<-gwr.multiscale(PURCHASE~FLOORSZ+PROF, data=londonhp, criterion="dCVR",kernel="gaussian",
adaptive=T, bws0=c(100, 100, 100), dMats=list(EUDM,EUDM,EUDM))
#Mixed GWR
res3<-gwr.multiscale(PURCHASE~FLOORSZ+PROF, data=londonhp, bws0=c(Inf, 100, 100, Inf),
               bw.seled=rep(T, 3),kernel="gaussian", dMats=list(EUDM,EUDM,EUDM))
#PSDM GWR
res4<- gwr.multiscale(PURCHASE~FLOORSZ+PROF, data=londonhp, kernel="gaussian", p.vals=c(1,2,3))</pre>
## End(Not run)
```

gwr.predict GWR used as a spatial predictor

## Description

This function implements basic GWR as a spatial predictor. The GWR prediction function is able to do leave-out-one predictions (when the observation locations are used for prediction) and predictions at a set-aside data set (when unobserved locations are used for prediction).

#### Usage

formula	Regression model formula of a formula object
data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame
	as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>

predictdata	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>
bw	bandwidth used in the weighting function, possibly calculated by bw.gwr;fixed (distance) or adaptive bandwidth(number of nearest neighbours)
kernel	function chosen as follows:
	gaussian: $wgt = exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
	tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat1	a pre-specified distance matrix between data points and prediction locations; if not given, it will be calculated by the given parameters
dMat2	a pre-specified sysmetric distance matrix between data points; if not given, it will be calculated by the given parameters
х	an object of class "gwrm.pred", returned by the function gwr.predict
	arguments passed through (unused)

A list of class "gwrm.pred":

GW.arguments	a list of geographically weighted arguments
SDF	a SpatialPointsDataFrame (may be gridded), or SpatialPolygonsDataFrame object (see package "sp"), or sf object (see package "sf") with GWR coefficients, predictions and prediction variances in its "data" slot.
this.call	the function call used.

#### Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

## References

Harris P, Fotheringham AS, Crespo R, Charlton M (2010) The use of geographically weighted regression for spatial prediction: an evaluation of models using simulated data sets. Mathematical Geosciences 42:657-680

Harris P, Juggins S (2011) Estimating freshwater critical load exceedance data for Great Britain using space-varying relationship models. Mathematical Geosciences 43: 265-292

#### gwr.robust

Harris P, Brunsdon C, Fotheringham AS (2011) Links, comparisons and extensions of the geographically weighted regression model when used as a spatial predictor. Stochastic Environmental Research and Risk Assessment 25:123-138

Gollini I, Lu B, Charlton M, Brunsdon C, Harris P (2015) GWmodel: an R Package for exploring Spatial Heterogeneity using Geographically Weighted Models. Journal of Statistical Software, 63(17):1-50

## Examples

gwr.robust Robust GWR model

## Description

This function implements two robust GWR models.

#### Usage

```
gwr.robust(formula, data, bw,filtered=FALSE, kernel = "bisquare", adaptive = FALSE, p = 2,
    theta = 0, longlat = F, dMat, F123.test = F, maxiter=20,cut.filter= 3,cut1=2,
    cut2=3,delta=1.0e-5, parallel.method = FALSE, parallel.arg = NULL)
```

formula	Regression model formula of a formula object
data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b>
bw	bandwidth used in the weighting function, possibly calculated by bw.gwr;fixed (distance) or adaptive bandwidth(number of nearest neighbours)
filtered	default FALSE, the automatic approach is used, if TRUE the filtered data approach is employed, as that described in Fotheringham et al. (2002 p.73-80)

kernel	function chosen as follows:
	gaussian: $wgt = exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
	tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist
F123.test	default FALSE, otherwise calculate F-test results (Leung et al. 2000)
maxiter	default 20, maximum number of iterations for the automatic approach
cut.filter	If filtered is TRUE, it will be used as the residual cutoff for filtering data; default cutoff is 3
cut1	default 2, first cutoff for the residual weighting function. $wr(e)=1$ if $ e <=cut1*sigma$
cut2	default 3, second cutoff for the residual weighting function. $wr(e)=(1-(lel-2)^2)^2$ if cut1*sigma <lel<cut2*sigma, and="" if="" lel="" wr(e)="0">=cut2*sigma; cut 1 and cut2 refer to the automatic approach</lel<cut2*sigma,>
delta	default 1.0e-5, tolerance of the iterative algorithm
parallel.method	
	FALSE as default, and the calibration will be conducted traditionally via the se- rial technique, "omp": multi-thread technique with the OpenMP API, "cluster": multi-process technique with the <b>parallel</b> package, "cuda": parallel computing technique with CUDA
parallel.arg	if parallel.method is not FALSE, then set the argument by following: if parallel.method is "omp", parallel.arg refers to the number of threads used, and its default value is the number of cores - 1; if parallel.method is "cluster", parallel.arg refers to the number of R sessions used, and its default value is the number of cores - 1; if parallel.method is "cuda", parallel.arg refers to the number of calibrations included in each group, but note a too large value may cause the overflow of GPU memory.

A list of class "gwrm":

GW.arguments	a list class object including the model fitting parameters for generating the report file
GW.diagnostic	a list class object including the diagnostic information of the model fitting
lm	an object of class inheriting from "lm", see lm.

#### gwr.robust

SDF	a SpatialPointsDataFrame (may be gridded), or SpatialPolygonsDataFrame object (see package "sp"), or sf object (see package "sf") integrated with fit.points,GWR coefficient estimates, y value,predicted values, coefficient standard errors and t-values in its "data" slot. Notably, E_weigts will be also included in the output SDF which represents the residual weighting when automatic approach is used; When the filtered approach is used, E_weight is a vector consisted of 0 and 1, where 0 means outlier to be excluded from calibration.
timings	starting and ending time.
this.call	the function call used.
Ftest.res	results of Leung's F tests when F123.test is TRUE.

## Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

#### References

Fotheringham S, Brunsdon, C, and Charlton, M (2002), Geographically Weighted Regression: The Analysis of Spatially Varying Relationships, Chichester: Wiley.

Harris P, Fotheringham AS, Juggins S (2010) Robust geographically weighed regression: a technique for quantifying spatial relationships between freshwater acidification critical loads and catchment attributes. Annals of the Association of American Geographers 100(2): 286-306

## Examples

```
## Not run:
data(DubVoter)
bw.a <- bw.gwr(GenEl2004~DiffAdd+LARent+SC1+Unempl+LowEduc+Age18_24</pre>
+Age25_44+Age45_64,
data=Dub.voter,approach="AICc",kernel="bisquare",adaptive=TRUE)
bw.a
gwr.res <- gwr.basic(GenEl2004~DiffAdd+LARent+SC1+Unempl+LowEduc+Age18_24</pre>
+Age25_44+Age45_64,
data=Dub.voter,bw=bw.a,kernel="bisquare",adaptive=TRUE,F123.test=TRUE)
print(gwr.res)
# Map of the estimated coefficients for LowEduc
names(gwr.res$SDF)
if(require("RColorBrewer"))
{
  mypalette<-brewer.pal(6,"Spectral")</pre>
  X11(width=10, height=12)
  spplot(gwr.res$SDF,"LowEduc",key.space = "right",
  col.regions=mypalette,at=c(-8,-6,-4,-2,0,2,4),
  main="Basic GW regression coefficient estimates for LowEduc")
}
# Robust GW regression and map of the estimated coefficients for LowEduc
rgwr.res <- gwr.robust(GenEl2004~DiffAdd+LARent+SC1+Unempl+LowEduc+Age18_24
+Age25_44+Age45_64, data=Dub.voter,bw=bw.a,kernel="bisquare",
adaptive=TRUE,F123.test=TRUE)
```

```
print(rgwr.res)
if(require("RColorBrewer"))
{
    X11(width=10,height=12)
    spplot(rgwr.res$SDF, "LowEduc", key.space = "right",
    col.regions=mypalette,at=c(-8,-6,-4,-2,0,2,4),
    main="Robust GW regression coefficient estimates for LowEduc")
}
### End(Not run)
```

gwr.scalable Scalable GWR

## Description

This function implements Scalable GWR for large dataset

## Usage

## Arguments

formula	Regression model formula of a formula object
data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>
bw.adapt	adaptive bandwidth (i.e. number of nearest neighbours) used for geographically weighting
kernel	Kernel function to calculate the spatial weights, but note only two continuous functions available:
	gaussian: $wgt = exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
polynomial	Degree of the polyunomial to approximate the kernel function, and default is 4.
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist
x	an object of class "scgwrm", returned by the function gwr.scalable
	arguments passed through (unused)

78

## gwr.t.adjust

#### Value

A list of class "scgwrm":

GW.arguments	a list class object including the model fitting parameters for generating the report file
GW.diagnostic	a list class object including the diagnostic information of the model fitting
lm	an object of class inheriting from "lm", see lm.
SDF	a SpatialPointsDataFrame (may be gridded), or SpatialPolygonsDataFrame object (see package "sp"), or sf object (see package "sf") integrated with fit.points,GWR coefficient estimates, y value,predicted values, coefficient standard errors and t-values in its "data" slot.
timings	starting and ending time.

## Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

## References

Murakami, D., N. Tsutsumida, T. Yoshida, T. Nakaya & B. Lu (2019) Scalable GWR: A linear-time algorithm for large-scale geographically weighted regression with polynomial kernels. arXiv:1905.00266.

#### Examples

```
## Not run:
require(spData)
data(boston)
boston <- boston.c
coordinates(boston) <- ~ LON + LAT
res <- gwr.scalable(formula = MEDV ~ CRIM + ZN + INDUS + CHAS + AGE, data = boston, bw.adapt = 100)
res
```

## End(Not run)

gwr.t.adjust

Adjust p-values for multiple hypothesis tests in basic GWR

## Description

Given a set of p-values from the pseudo t-tests of basic GWR outputs, this function returns adjusted p-values using: (a) Bonferroni, (b) Benjamini-Hochberg, (c) Benjamini-Yekutieli and (d) Fotheringham-Byrne procedures.

#### Usage

gwr.t.adjust(gwm.Obj)

#### Arguments

gwm.Obj

an object of class "gwrm", returned by the function gwr.basic

## Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

## References

Byrne, G., Charlton, M. and Fotheringham, S., 2009. Multiple dependent hypothesis tests in geographically weighted regression. In: Lees, B. and Laffan, S. eds. 10th International conference on geocomputation. Sydney.

gwr.write

Write the GWR results into files

#### Description

This function writes the calibration result of function gwr.basic to a text file and shape files

## Usage

```
gwr.write(x,fn="GWRresults")
gwr.write.shp(x,fn="GWRresults")
```

## Arguments

х	an object of class "gwrm", returned by the function gwr.basic
fn	file name for the written results, by default the output files can be found in the working directory, "GWRresults.txt", "GWRresults(.shp, .shx, .dbf)"

## Note

The projection file is missing for the writen shapefiles.

The functions "writeGWR" and "writeGWR.shp" (in the early versions of GWmodel) have been renamed respectively as "gwr.write" and "gwr.write.shp", while the old names are still kept valid.

#### Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

## Description

gwss

This function calculates basic and robust GWSS. This includes geographically weighted means, standard deviations and skew. Robust alternatives include geographically weighted medians, interquartile ranges and quantile imbalances. This function also calculates basic geographically weighted covariances together with basic and robust geographically weighted correlations.

## Usage

data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b>
summary.locat	a Spatial*DataFrame object for providing summary locations, i.e. SpatialPoints- DataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b>
vars	a vector of variable names to be summarized
bw	bandwidth used in the weighting function
kernel	function chosen as follows:
	gaussian: $wgt = exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
	tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise
adaptive	if TRUE calculate an adaptive kernel where the bandwidth (bw) corresponds to the number of nearest neighbours (i.e. adaptive distance); default is FALSE, where a fixed kernel is found (bandwidth is a fixed distance)
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist
quantile	if TRUE, median, interquartile range, quantile imbalance will be calculated
x	an object of class "gwss", returned by the function gwss
	arguments passed through (unused)

A list of class "lss":

SDF	a SpatialPointsDataFrame (may be gridded) or SpatialPolygonsDataFrame object (see package "sp") with local means,local standard deviations,local variance, local skew,local coefficients of variation, local covariances, local correlations (Pearson's), local correlations (Spearman's), local medians, local interquartile ranges, local quantile imbalances and coordinates.
	other information for reporting

#### Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

## References

Fotheringham S, Brunsdon, C, and Charlton, M (2002), Geographically Weighted Regression: The Analysis of Spatially Varying Relationships, Chichester: Wiley.

Brunsdon C, Fotheringham AS, Charlton ME (2002) Geographically weighted summary statistics - a framework for localised exploratory data analysis. Computers, Environment and Urban Systems 26:501-524

Harris P, Clarke A, Juggins S, Brunsdon C, Charlton M (2014) Geographically weighted methods and their use in network re-designs for environmental monitoring. Stochastic Environmental Research and Risk Assessment 28: 1869-1887

# Examples

```
## Not run:
data(EWHP)
data(EWOutline)
head(ewhp)
houses.spdf <- SpatialPointsDataFrame(ewhp[, 1:2], ewhp)</pre>
localstats1 <- gwss(houses.spdf, vars = c("PurPrice", "FlrArea"), bw = 50000)</pre>
head(data.frame(localstats1$SDF))
localstats1
##A function for mapping data
if(require("RColorBrewer"))
{
   quick.map <- function(spdf,var,legend.title,main.title)</pre>
   {
     x <- spdf@data[,var]</pre>
     cut.vals <- pretty(x)</pre>
     x.cut <- cut(x,cut.vals)</pre>
     cut.levels <- levels(x.cut)</pre>
     cut.band <- match(x.cut,cut.levels)</pre>
     colors <- brewer.pal(length(cut.levels), "YlOrRd")</pre>
     colors <- rev(colors)</pre>
     par(mar=c(1,1,1,1))
     plot(ewoutline,col="olivedrab",bg="lightblue1")
     title(main.title)
```

```
plot(spdf,add=TRUE,col=colors[cut.band],pch=16)
     legend("topleft",cut.levels,col=colors,pch=16,bty="n",title=legend.title)
 }
 quick.map(localstats1$SDF, "PurPrice_LM", "1000's Uk Pounds",
  "Geographically Weighted Mean")
 par(mfrow = c(1, 2))
 quick.map(localstats1$SDF, "PurPrice_LSKe", "Skewness Level", "Local Skewness")
 quick.map(localstats1$SDF, "PurPrice_LSD", "1000's Pounds", "Local Standard Deviation")
 #Exploring Non-Stationarity of Relationships
 quick.map(localstats1$SDF, "Corr_PurPrice.FlrArea", expression(rho),
  "Geographically Weighted Pearson Correlation")
 #Robust, Quantile Based Local Summary Statistics
 localstats2 <- gwss(houses.spdf, vars = c("PurPrice", "FlrArea"),</pre>
 bw = 50000, quantile = TRUE)
 quick.map(localstats2$SDF, "PurPrice_Median", "1000 UK Pounds",
  "Geographically Weighted Median House Price")
}
## End(Not run)
```

gwss.montecarlo Monte Carlo (randomisation) test for gwss

## Description

This function implements Monte Carlo (randomisation) tests for the GW summary statistics found in gwss.

#### Usage

data	a Spatial*DataFrame, i.e. SpatialPointsDataFrame or SpatialPolygonsDataFrame as defined in package <b>sp</b> , or a sf object defined in package <b>sf</b>
vars	a vector of variable names to be summarized
bw	bandwidth used in the weighting function
kernel	function chosen as follows:
	gaussian: $wgt = exp(5*(vdist/bw)^2);$
	exponential: wgt = exp(-vdist/bw);
	bisquare: wgt = (1-(vdist/bw)^2)^2 if vdist < bw, wgt=0 otherwise;
	tricube: wgt = (1-(vdist/bw)^3)^3 if vdist < bw, wgt=0 otherwise;
	boxcar: wgt=1 if dist < bw, wgt=0 otherwise

adaptive	if TRUE calulate the adaptive kernel, and bw correspond to the number of near- est neighbours, default is FALSE.
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
dMat	a pre-specified distance matrix, it can be calculated by the function gw.dist
quantile	if TRUE, median, interquartile range, quantile imbalance will be calculated
nsim	default 99, the number of randomisations

test	probability of the test statistics of the GW summary statistics; if p<0.025 or if
	p>0.975 then the true local summary statistics can be said to be significantly
	different (at the 0.95 level) to such a local summary statistics found by chance.

## Note

The function "montecarlo.gwss" (in the early versions of GWmodel) has been renamed as "gwss.montecarlo", while the old name is still kept valid.

#### Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

#### References

Fotheringham S, Brunsdon, C, and Charlton, M (2002), Geographically Weighted Regression: The Analysis of Spatially Varying Relationships, Chichester: Wiley.

Brunsdon C, Fotheringham AS, Charlton ME (2002) Geographically weighted summary statistics - a framework for localised exploratory data analysis. Computers, Environment and Urban Systems 26:501-524

Harris P, Brunsdon C (2010) Exploring spatial variation and spatial relationships in a freshwater acidification critical load data set for Great Britain using geographically weighted summary statistics. Computers & Geosciences 36:54-70

#### Examples

LondonBorough

## Description

Outline (SpatialPolygonsDataFrame) of London boroughs for the LondonHP data.

## Usage

data(LondonBorough)

## Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

LondonHP

London house price data set (SpatialPointsDataFrame)

#### Description

A house price data set with 18 hedonic variables for London in 2001.

#### Usage

data(LondonHP)

#### Format

A SpatialPointsDataFrame object (proj4string set to "+init=epsg:27700 +datum=OSGB36").

The "data" slot is a data frame with 372 observations on the following 21 variables.

X a numeric vector, X coordinate

Y a numeric vector, Y coordinate

**PURCHASE** a numeric vector, the purchase price of the property

FLOORSZ a numeric vector, floor area of the property in square metres

- **TYPEDETCH** a numeric vector, 1 if the property is detached (i.e. it is a stand-alone house), 0 otherwise
- **TPSEMIDTCH** a numeric vector, 1 if the property is semi detached, 0 otherwise
- **TYPETRRD** a numeric vector, 1 if the property is in a terrace of similar houses (commonly referred to as a 'row house' in the USA), 0 otherwise
- **TYPEBNGLW** a numeric vector, if the property is a bungalow (i.e. it has only one floor), 0 otherwise
- TYPEFLAT a numeric vector, if the property is a flat (or 'apartment' in the USA), 0 otherwise

BLDPWW1 a numeric vector, 1 if the property was built prior to 1914, 0 otherwise

BLDPOSTW a numeric vector, 1 if the property was built between 1940 and 1959, 0 otherwise

BLD60S a numeric vector, 1 if the property was built between 1960 and 1969, 0 otherwise

**BLD70S** a numeric vector, 1 if the property was built between 1970 and 1979, 0 otherwise

**BLD80S** a numeric vector, 1 if the property was built between 1980 and 1989, 0 otherwise

**BLD90S** a numeric vector, 1 if the property was built between 1990 and 2000, 0 otherwise

BATH2 a numeric vector, 1 if the property has more than 2 bathrooms, 0 otherwise

GARAGE a numeric vector, 1 if the house has a garage, 0 otherwise

**CENTHEAT** a numeric vector, 1 if the house has central heating, 0 otherwise

BEDS2 a numeric vector, 1 if the property has more than 2 bedrooms, 0 otherwise

- **UNEMPLOY** a numeric vector, the rate of unemployment in the census ward in which the house is located
- **PROF** a numeric vector, the proportion of the workforce in professional or managerial occupations in the census ward in which the house is located

#### Author(s)

Binbin Lu <binbinlu@whu.edu.cn>

#### References

Fotheringham, A.S., Brunsdon, C., and Charlton, M.E. (2002), Geographically Weighted Regression: The Analysis of Spatially Varying Relationships, Chichester: Wiley.

Lu, B, Charlton, M, Harris, P, Fotheringham, AS (2014) Geographically weighted regression with a non-Euclidean distance metric: a case study using hedonic house price data. International Journal of Geographical Information Science 28(4): 660-681

## Examples

```
data(LondonHP)
data(LondonBorough)
ls()
plot(londonborough)
plot(londonhp, add=TRUE)
```

st.dist

Spatio-temporal distance matrix calculation

#### Description

Calculate a distance vector(matrix) between any GW model calibration point(s) and the data points.

# st.dist

# Usage

# Arguments

dp.locat	a numeric matrix of two columns giving the coordinates of the data points
rp.locat	a numeric matrix of two columns giving the coordinates of the GW model calibration points
obs.tv	a vector of time tags for each observation, which could be numeric or of <b>POSIXIt</b> class
reg.tv	a vector of time tags for each regression location, which could be numeric or of <b>POSIXIt</b> class
focus	an integer, indexing to the current GW model point, if focus=0, all the distances between all the GW model calibration points and data points will be calculated and a distance matrix will be returned; if 0 <focus<length(rp.locat), 'focus'th="" a="" and="" be="" between="" calculated="" data="" distance="" distances="" gw="" model="" points="" returned<="" td="" the="" then="" vector="" will=""></focus<length(rp.locat),>
р	the power of the Minkowski distance, default is 2, i.e. the Euclidean distance
theta	an angle in radians to rotate the coordinate system, default is 0
longlat	if TRUE, great circle distances will be calculated
lamda	an parameter between 0 and 1 for calculating spatio-temporal distance
t.units	character string to define time unit
ksi	an parameter between 0 and PI for calculating spatio-temporal distance, see de- tails in Wu et al. (2014)
s.dMat	
S. una c	a predifined spatial distance matrix for calculating spatio-temporal distances

# Value

Returns a numeric spatio-temporal distance matrix or vector; or a matrix with its rows corresponding to the observations and its columns corresponds to the calibration points.

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USelect

*Results of the 2004 US presidential election at the county level (SpatialPolygonsDataFrame)* 

### Description

Results of the 2004 US presidential election at the county level, together with five socio-economic (census) variables. This data can be used with GW Discriminant Analysis.

## Usage

data(USelect)

## Format

A SpatialPolygonsDataFrame with 3111 electoral divisions on the following 6 variables.

winner Categorical variable with three classes: i) Bush, ii) Kerry and iii) Borderline (supporting ratio for a candidate ranges from 0.45 to 0.55)

unemploy percentage unemployed

pctcoled percentage of adults over 25 with 4 or more years of college education

PEROVER65 percentage of persons over the age of 65

pcturban percentage urban

WHITE percentage white

#### References

Robinson, A. C. (2013). Geovisualization of the 2004 Presidential Election. In: NATIONAL IN-STITUTES OF HEALTH, P. S. U. (ed.). Penn State.

Foley, P. & Demsar, U. (2012). Using geovisual analytics to compare the performance of geographically weighted discriminant analysis versus its global counterpart, linear discriminant analysis. International Journal of Geographical Information Science, 27, 633-661.

#### Examples

data(USelect) ls()

# Index

\* Cross-validation score ggwr.cv,20 \* Dublin Voter turnout DubVoter, 14 \* England-Wales outline EWOutline, 16 \* GTWR bw.gtwr,6 gtwr, 22 \* GW tools gw.dist, 24 gw.pcplot, 26 gw.weight, 27 \* GWDA bw.gwda,7 gwda, 28 \* GWPCA bw.gwpca, 9 gwpca, 30 gwpca.check.components, 33 gwpca.cv, 34 gwpca.cv.contrib, 35 gwpca.glyph.plot, 36 gwpca.montecarlo.1,36 gwpca.montecarlo.2,38 \* GWPCP gw.pcplot, 26 \* GWR-LCR bw.gwr.lcr, 11 gwr.lcr, 53 gwr.lcr.cv, 56 gwr.lcr.cv.contrib, 57 \* GWR bw.gwr, 10 gwr.basic,40 gwr.bootstrap, 44 gwr.collin.diagno,48 gwr.cv, 49gwr.cv.contrib, 50

gwr.mink.approach, 58 gwr.mink.matrixview, 60 gwr.mink.pval, 61 gwr.model.selection, 64 gwr.model.sort, 66 gwr.model.view, 67 gwr.montecarlo,68 gwr.predict, 73 gwr.t.adjust,79 gwr.write, 80 \* GWSS bw.gwss.average, 13 gwss, 81 gwss.montecarlo, 83 \* Georgia census Georgia, 16 \* Georgia counties GeorgiaCounties, 17 \* Heteroskedastic GWR gwr.hetero, 52 \* London Boroughs LondonBorough, 85 \* Minkowski approach view gwr.mink.matrixview, 60 \* Minkowski approach gwr.mink.approach, 58 gwr.mink.pval, 61 \* Monte Carlo test gwr.montecarlo,68 \* Monte Carlo gwpca.montecarlo.1,36 gwpca.montecarlo.2,38 gwss.montecarlo,83 **\* PSDM GWR** gwr.multiscale, 69 \* Scalable GWR gwr.scalable, 78 \* US presidential election USelect, 88

\* bandwidth selection bw.ggwr,4 bw.gtwr,6 bw.gwda,7 bw.gwpca, 9 bw.gwr, 10 bw.gwr.lcr, 11 bw.gwss.average, 13 \* bootstrap method gwr.bootstrap,44 \* collinearity diagnostics gwr.collin.diagno, 48 \* cross-validation score gwpca.cv, 34 gwr.cv,49 gwr.lcr.cv, 56 \* data DubVoter, 14 EWHP, 15 EWOutline, 16 Georgia. 16 GeorgiaCounties, 17 LondonBorough, 85 LondonHP, 85 USelect, 88 \* distance gw.dist, 24 \* generalised GWR bw.ggwr,4 ggwr.basic, 18 ggwr.cv, 20 ggwr.cv.contrib, 21 \* glyph plot interaction gwpca.check.components, 33 \* glyph plot gwpca.glyph.plot, 36 \* gtwr st.dist,86 \* house price EWHP, 15 LondonHP, 85 \* mixed GWR gwr.mixed, 62 \* model selection gwr.model.selection, 64 gwr.model.sort, 66 gwr.model.view, 67 \* multiscale GWR

gwr.mixed, 62 gwr.multiscale, 69 \* p-values adjustment gwr.t.adjust, 79 \* package GWmodel-package, 3 \* point-wise cross-validation scores gwpca.cv.contrib, 35 gwr.cv.contrib, 50 gwr.lcr.cv.contrib, 57 \* point-wise cross-validation score ggwr.cv.contrib, 21 \* predictor gwr.predict, 73 \* results writing gwr.write, 80 \* robust GWR gwr.robust,75 \* weight gw.weight, 27 AICc (gwr.model.selection), 64 AICc1 (gwr.scalable), 78 AICc\_rss (gwr.model.selection), 64 AICc\_rss1 (bw.gwr), 10 bias.bs(gwr.bootstrap), 44 bw.ggwr,4 bw.gtwr,6 bw.gwda, 7 bw.gwpca, 9, 29, 30, 37 bw.gwr, 10, 23, 27, 40, 52, 59, 61, 63, 65, 68, 74.75 bw.gwr.lcr, 11 bw.gwr1 (gwr.mink.approach), 58 bw.gwr3 (gwr.bootstrap), 44 bw.gwss.average, 13 check.components (gwpca.check.components), 33 ci.bs(gwr.bootstrap),44 Ci\_mat (gwr.basic), 40 confusion.matrix (gwda), 28 dist, 25 Dub.voter (DubVoter), 14 DubVoter, 14

e\_vec(bw.gwr), 10

## INDEX

ehat (gwr.model.selection), 64 EWHP, 15, 16 ewhp (EWHP), 15 EWOutline, 16 ewoutline (EWOutline), 16 extract.mat(gwr.model.selection), 64 F1234.test (gwr.basic), 40 fitted (bw.gwr), 10 formula, 5-7, 10, 12, 18, 23, 28, 40, 45, 46, 48, 52, 53, 59, 61, 63, 68, 70, 73, 75, 78 Gedu.counties (GeorgiaCounties), 17 Gedu.df (Georgia), 16 Generate.formula(gwr.model.selection), 64 generate.lm.data(gwr.bootstrap), 44 Georgia, 16 GeorgiaCounties, 17 ggwr.aic(bw.ggwr),4 ggwr.basic, 18 ggwr.cv, 20 ggwr.cv.contrib, 21 glm, 19 glyph.plot(gwpca.glyph.plot), 36 gold (bw.gwr), 10 grouping.xy (gwda), 28 gtwr, 22 gtwr.aic(bw.gtwr),6 gtwr.cv (bw.gtwr), 6 gw.average.cv(bw.gwss.average), 13 gw.dist, 5, 8, 9, 11-13, 19, 21, 22, 24, 26, 29, 31, 34, 35, 37, 39, 40, 45, 48, 50–52, 54, 57, 58, 63, 65, 68, 76, 78, 81, 84 gw.fitted (gwr.model.selection), 64 gw.mean.cv (bw.gwss.average), 13 gw.median.cv(bw.gwss.average), 13 gw.pcplot, 26 gw.reg1 (gwr.predict), 73 gw.weight, 27 gw\_BIC (bw.gwr), 10 gw\_cv\_all (bw.gwr), 10 gw\_cv\_all\_cuda (bw.gwr), 10 gw\_cv\_all\_omp (gwr.basic), 40 gw\_dist (gw.dist), 24 gw\_fitted (gwr.multiscale), 69 gw\_local\_r2(gwr.basic), 40 gw\_reg(gwr.basic), 40

gw\_reg\_1 (gwr.basic), 40 gw\_reg\_2(gwr.basic), 40 gw\_reg\_all (gwr.basic), 40 gw\_reg\_all\_cuda (gwr.basic), 40 gw\_reg\_all\_omp (gwr.basic), 40 gw\_weight(gw.weight), 27 gw\_weight\_mat(gw.weight), 27 gw\_weight\_vec(gw.weight), 27 gwda, 28, 29 GWmodel (GWmodel-package), 3 GWmodel-package, 3 gwpca, 30, 31, 33, 36 gwpca.check.components, 33 gwpca.cv, 34gwpca.cv.contrib, 35 gwpca.glyph.plot, 33, 36 gwpca.montecarlo.1, 36, 37 gwpca.montecarlo.2, 37, 38 gwr.aic(bw.gwr), 10 gwr.aic1(gwr.mink.approach), 58 gwr.backfit(gwr.multiscale), 69 gwr.basic, 40, 41, 80 gwr.binomial(ggwr.basic), 18 gwr.bootstrap, 44, 45 gwr.collin.diagno, 48 gwr.cv, 49gwr.cv.contrib, 50 gwr.cv1 (gwr.mink.approach), 58 gwr.fitted(ggwr.basic), 18 gwr.generalised, 19 gwr.generalised(ggwr.basic), 18 gwr.hetero, 52 gwr.lcr, 11, 53, 54 gwr.lcr.cv, 56 gwr.lcr.cv.contrib, 57 gwr.mink.approach, 58, 60 gwr.mink.matrixview, 60 gwr.mink.pval, 61 gwr.mixed, 62, 72 gwr.model.selection, 64, 66, 67 gwr.model.sort, 66, 66, 67 gwr.model.view, 66, 67, 67 gwr.montecarlo,68 gwr.multiscale, 69, 71, 72 gwr.poisson(ggwr.basic), 18 gwr.predict, 73, 74 gwr.q(gwr.mixed), 62 gwr.q2(gwr.multiscale), 69

## INDEX

```
gwr.robust,75
gwr.scalable, 78, 78
gwr.t.adjust, 79
gwr.write, 80
gwr_diag (gwr.basic), 40
gwr_diag1 (gwr.scalable), 78
gwr_mixed_2 (gwr.mixed), 62
gwr_mixed_trace (gwr.mixed), 62
gwr_q(gwr.mixed), 62
gwrt.err (gwr.bootstrap), 44
gwrt.lag(gwr.bootstrap), 44
gwrt.mlr(gwr.bootstrap), 44
gwrt.sma (gwr.bootstrap), 44
gwrtvar (gwr.bootstrap), 44
gwss, 81, 81, 83
gwss.montecarlo, 83
```

```
list, 19, 70, 76
lm, 23, 41, 72, 76, 79
local.corr (gwss), 81
LondonBorough, 85
londonborough (LondonBorough), 85
LondonHP, 85, 85
londonhp (LondonHP), 85
```

```
par, 26
parametric.bs (gwr.bootstrap), 44
plot.mcsims (gwpca.montecarlo.1), 36
plot.pvlas (gwr.mink.pval), 61
POSIX1t, 6, 23, 87
princomp, 31
print.ggwrm (ggwr.basic), 18
print.gtwrm (gtwr), 22
```

print.gwda(gwda), 28 print.gwpca(gwpca), 30 print.gwrbsm(gwr.bootstrap), 44 print.gwrlcr(gwr.lcr), 53 print.gwrm(gwr.basic), 40 print.gwrm.pred(gwr.predict), 73 print.gwss(gwss), 81 print.mgwr(gwr.mixed), 62 print.multiscalegwr(gwr.multiscale), 69 print.scgwrm(gwr.scalable), 78 pval.bs(gwr.bootstrap), 44

ridge.lm(gwr.lcr), 53
robustSvd(gwpca), 30
rss(gwr.model.selection), 64
rwpca(gwpca), 30

scgwr\_loocv (gwr.scalable), 78 scgwr\_pre (gwr.scalable), 78 scgwr\_reg (gwr.scalable), 78 se.bs (gwr.bootstrap), 44 splitx (gwda), 28 st.dist, 23, 86

```
ti.dist(gtwr), 22
ti.distm(gtwr), 22
ti.distv(gtwr), 22
ti.distv(gtwr), 22
trhat2(gwr.basic), 40
```

USelect, 88 USelect2004(USelect), 88

```
vector, 70
```

wlda (gwda), 28
wlda.cr (bw.gwda), 7
wmean (gwda), 28
wpca (gwpca), 30
wprior (gwda), 28
wqda (gwda), 28
wqda.cr (bw.gwda), 7
writeGWR (gwr.write), 80
wt.median (gwpca), 30
wvarcov (gwda), 28

## 92