# Package 'GECal'

January 20, 2025

Type Package

Title Generalized Entropy Calibration

Version 0.1.5

**Description** Generalized Entropy Calibration produces calibration weights using generalized entropy as the objective function for optimization. This approach, as implemented in the 'GECal' package, is based on Kwon, Kim, and Qiu (2024) <doi:10.48550/arXiv.2404.01076>. Unlike traditional methods, 'GECal' incorporates design weights into the constraints to maintain design consistency, rather than including them in the objective function itself.

**Encoding** UTF-8

URL https://github.com/yonghyun-K/GECal

BugReports https://github.com/yonghyun-K/GECal/issues

**Depends** R (>= 2.10.0)

LazyData true

Imports nleqslv

Suggests sampling

RoxygenNote 7.3.2

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NeedsCompilation no

Author Yonghyun Kwon [aut, cre] (<https://orcid.org/0000-0001-9923-6790>), Jae Kwang Kim [aut] (<https://orcid.org/0000-0002-0246-6029>), Yumou Qiu [aut] (<https://orcid.org/0000-0003-4846-1263>)

Maintainer Yonghyun Kwon <yhkwon@kma.ac.kr>

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# estimate

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estimate

Performing statistical inference after calibration

# Description

estimate performs statistical inference after calibration.

#### Usage

```
estimate(formula, data = NULL, calibration, pimat = NULL)
```

# Arguments

formula	An object of class "formula" specifying the calibration model.
data	An optional data frame containing the variables in the model (specified by formula).
calibration	An object of class "calibration", generated by GECalib.
pimat	An optional matrix contatining the joint inclusion probability matrix used for variance estimation.

#### Value

A list of class estimation including the point estimates and its standard error.

# References

Kwon, Y., Kim, J., & Qiu, Y. (2024). Debiased calibration estimation using generalized entropy in survey sampling. Arxiv preprint <a href="https://arxiv.org/abs/2404.01076">https://arxiv.org/abs/2404.01076</a>>

Deville, J. C., and Särndal, C. E. (1992). Calibration estimators in survey sampling. Journal of the American statistical Association, 87(418), 376-382.

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#### estimate

#### Examples

set.seed(11)

```
N = 10000
x = data.frame(x1 = rnorm(N, 2, 1), x2= runif(N, 0, 4))
pi = pt((-x[,1] / 2 - x[,2] / 2), 3);
pi = ifelse(pi >.7, .7, pi)
delta = rbinom(N, 1, pi)
Index_S = (delta == 1)
pi_S = pi[Index_S]; d_S = 1 / pi_S
x_S = x[Index_S,,drop = FALSE]
# pimat = diag(d_S^2 - d_S) / N^2 # 1 / pi_i * (1 - 1 / pi_i)
e = rnorm(N, 0, 1)
y = x[,1] + x[,2] + e;
y_S = y[Index_S] # plot(x_S, y_S)
calibration0 <- GECal::GEcalib(~ 1, dweight = d_S, data = x_S,</pre>
                                const = N,
                                entropy = "SL", method = "DS")
GECal::estimate(y_S \sim 1, calibration = calibration0)$estimate # Hajek estimator
# sum(y_S * d_S) * N / sum(d_S)
calibration <- GECal::GEcalib(~ 0, dweight = d_S, data = x_S,</pre>
const = numeric(0),
entropy = "SL", method = "DS")
GECal::estimate(y_S \sim 1, calibration = calibration)$estimate # HT estimator
calibration1 <- GECal::GEcalib(~ ., dweight = d_S, data = x_S,</pre>
                                const = colSums(cbind(1, x)),
                                entropy = "ET", method = "DS")
GECal::estimate(y_S ~ 1, calibration = calibration1)$estimate
calibration2 <- GECal::GEcalib(~ ., dweight = d_S, data = x_S,</pre>
                                const = colSums(cbind(1, x)),
                                entropy = "ET", method = "GEC0")
GECal::estimate(y_S ~ 1, calibration = calibration2)$estimate
calibration3 <- GECal::GEcalib(~ . + g(d_S), dweight = d_S, data = x_S,</pre>
                                const = colSums(cbind(1, x, log(1 / pi))),
                                entropy = "ET", method = "GEC")
GECal::estimate(y_S ~ 1, calibration = calibration3)$estimate
calibration4 <- GECal::GEcalib(~ . + g(d_S), dweight = d_S, data = x_S,</pre>
                                const = colSums(cbind(1, x, NA)),
                                entropy = "ET", method = "GEC")
GECal::estimate(y_S ~ 1, calibration = calibration4)$estimate
calibration5 <- GECal::GEcalib(~ . + g(d_S), dweight = d_S, data = x_S,
                                const = colSums(cbind(1, x, NA)),
                                entropy = "ET", method = "GEC", K_alpha = "log")
GECal::estimate(y_S ~ 1, calibration = calibration5)$estimate
```

Debiasing covariate for GECalib

# Description

g

It returns the debiasing covariate, which is equivalent to the first order derivatie of the generalized entropy G.

# Usage

g(x, entropy = NULL, del = NULL)

# Arguments

х	A vector of design weights
entropy	An optional data frame containing the variables in the model (specified by formula).
del	The optional vector for threshold ( $\delta$ ) when entropy == "PH".

# Value

A vector of debiasing covariate.

# Examples

```
set.seed(11)
N = 10000
x = data.frame(x1 = rnorm(N, 2, 1), x2= runif(N, 0, 4))
pi = pt((-x[,1] / 2 - x[,2] / 2), 3);
pi = ifelse(pi >.7, .7, pi)
g_EL <- g(1 / pi, entropy = 1)
g_ET <- g(1 / pi, entropy = 0)
g_EL <- g(1 / pi, entropy = -1)</pre>
```

GEcalib

# Description

GEcalib computes the calibration weights. Generalized entropy calibration weights maximize the generalized entropy:

$$H(\boldsymbol{\omega}) = -\sum_{i \in A} G(\omega_i),$$

subject to the calibration constraints  $\sum_{i \in A} \omega_i z_i = \sum_{i \in U} z_i$ , where A denotes the sample index, and U represents the population index. The auxiliary variables, whose population totals are known, are defined as  $z_i^T = (x_i^T, g(d_i))$ , where g is the first-order derivative of the gerenalized entropy G, and  $d_i$  is the design weight for each sampled unit  $i \in A$ .

# Usage

```
GEcalib(
  formula,
  dweight,
  data = NULL,
  const,
  method = c("GEC", "GEC0", "DS"),
  entropy = c("SL", "EL", "ET", "CE", "HD", "PH"),
  weight.scale = 1,
  G.scale = 1,
  K_alpha = NULL,
  is.total = TRUE,
  del = NULL
)
```

#### Arguments

formula	An object of class "formula" specifying the calibration model.
dweight	A vector of sampling weights.
data	An optional data frame containing the variables in the model (specified by formula).
const	A vector used in the calibration constraint for population totals( or means).
method	The method to be used in calibration. See "Details" for more information.
entropy	The generalized entropy used in calibration, which can be either a numeric value or a string. If numeric, entropy represents the order of Renyi's entropy, where $G(\omega) = r^{-1}(r+1)^{-1}\omega^{r+1}$ if $r \neq 0, -1$ . If a string, valid options include: "SL" (Squared-loss), "EL" (Empirical Likelihood), "ET" (Exponential Tilting), "CE" (Cross-Entropy), "HD" (Hellinger Distance), and "PH" (Pseudo-Huber). See "Summary" for details.
weight.scale	Positive scaling factor for the calibration weights $\omega_i$ . Asymptotics justify setting weight.scale to the finite population correction ( $fpc = n/N$ ).

G.scale	Positive scaling factor for the generalized entropy function $G$ . Asymptotics justify setting G. scale to the variance of the error term in a linear super-population model.
K_alpha	The $K$ function used in joint optimization when the const of the debiasing covariate $g(d_i)$ is not available. K_alpha can be NULL, "log", or custom functions. See "Details".
is.total	Logical, TRUE if sum(const[1]) equals the population size.
del	The optional threshold ( $\delta$ ) used when Pseudo-Huber (PH) entropy is selected. del = quantile(dweight, 0.75) if not specified.

# Details

The GEcal object returns the calibration weights and necessary information for estimating population totals(or mean).

The terms to the right of the ~ symbol in the formula argument define the calibration constraints. When method == "GEC", the debiasing covariate g(dweight) must be included in the formula. If the population total(mean) of g(dweight) is unavailable, const that corresponds to g(dweight) can be set to NA. In this case, GECalib performs joint optimization over both the calibration weights  $\omega_i$  and the missing value of const.

The length of the const vector should match the number of columns in the model.matrix generated by formula. Additionally, the condition number of the model.matrix must exceed .Machine\$double.eps to ensure its invertibility.

Both weight.scale and G.scale are positive scaling factors used for calibration. Note that weight.scale is not supported when method == "DS".

Let  $q_i$  be the scaling factor for the generalized entropy function G, and  $\phi_i$  be the scaling factor for the calibration weights  $\omega_i$ .

If method == "GEC", GEcalib minimizes the negative entropy:

$$\sum_{i \in A} q_i G(\phi_i \omega_i)$$

with respect to  $\boldsymbol{\omega}$  subject to the calibration constraints  $\sum_{i \in A} \omega_i \boldsymbol{z}_i = \sum_{i \in U} \boldsymbol{z}_i$ , where  $\boldsymbol{z}_i^T = (\boldsymbol{x}_i^T, q_i \phi_i g(\phi_i d_i))$ , A denotes the sample index, and U represents the population index.

If method == "GEC", but an element of const corresponding to the debiasing covariate  $g(d_i)$  is NA, GEcalib minimizes the negative adjusted entropy:

$$\sum_{i \in A} q_i G(\phi_i \omega_i) - K(\alpha),$$

with respect to  $\boldsymbol{\omega}$  and  $\alpha$  subject to the calibration constraints  $\sum_{i \in A} \omega_i(\boldsymbol{x}_i^T, q_i \phi_i g(\phi_i d_i)) = (\sum_{i \in U} \boldsymbol{x}_i, \alpha)$ , where the solution  $\hat{\alpha}$  is an estimate of population total for  $g(d_i)$ . Examples of  $K(\alpha)$  includes  $K(\alpha) = \alpha$  when K\_alpha == NULL, and

$$K(\alpha) = \left(\sum_{i \in A} d_i g(d_i) + N\right) \log \left| \frac{1}{N} \sum_{i \in A} q_i \phi_i \omega_i g(\phi_i \omega_i) + 1 \right|$$

when K\_alpha == "log".

#### GEcalib

$$\sum_{i \in A} q_i G(\phi_i \omega_i) - q_i \phi_i \omega_i g(\phi_i \omega_i)$$

with respect to  $\boldsymbol{\omega}$  subject to the calibration constraints  $\sum_{i \in A} \omega_i \boldsymbol{x}_i = \sum_{i \in U} \boldsymbol{x}_i$ . If method == "DS", GEcalib minimizes the divergence between  $\boldsymbol{\omega}$  and  $\boldsymbol{d}$ :

$$\sum_{i \in A} q_i d_i \tilde{G}(\omega_i/d_i)$$

with respect to  $\boldsymbol{\omega}$  subject to the calibration constraints  $\sum_{i \in A} \omega_i \boldsymbol{x}_i = \sum_{i \in U} \boldsymbol{x}_i$ . When method == "DS", weight.scale, the scaling factor for the calibration weights  $\phi_i$ , is not applicable.

Examples of G and  $\tilde{G}$  are given in "Summary".

GEC

#### Value

A list of class calibration including the calibration weights and data needed for estimation.

#### Summary

The table below provides a comparison between the GEC and DS methods.

$$\begin{split} \min_{\boldsymbol{\omega}} \left(-H(\boldsymbol{\omega})\right) &= \sum_{i \in A} G(\omega_i) \\ \text{s.t.} \sum_{i \in A} \omega_i(\boldsymbol{x}_i^T, g(d_i)) &= \sum_{i \in U} (\boldsymbol{x}_i^T, g(d_i)) \\ G(\boldsymbol{\omega}) &= \begin{cases} \frac{1}{r(r+1)} \omega^{r+1} & r \neq 0, -1 \\ \omega \log \omega - \omega & r = 0(\text{ET}) \\ -\log \omega & r = -1(\text{EL}) \end{cases} \quad \tilde{G}(\boldsymbol{\omega}) &= \begin{cases} \frac{1}{r(r+1)} (\omega^{r+1} - (r+1)\omega + r) & r \neq 0, -1 \\ \omega \log \omega - \omega + 1 & r = 0(\text{ET}) \\ -\log \omega + \omega - 1 & r = -1(\text{EL}) \end{cases} \end{split}$$

DS

If method == "GEC", further examples include

$$G(\omega) = (\omega - 1)\log(\omega - 1) - \omega\log\omega$$

when entropy == "CE", and

$$G(\omega) = \delta^2 \left( 1 + (\omega/\delta)^2 \right)^{1/2}$$

for a threshold  $\delta$  when entropy == "PH".

#### Author(s)

Yonghyun Kwon

#### References

Kwon, Y., Kim, J., & Qiu, Y. (2024). Debiased calibration estimation using generalized entropy in survey sampling. Arxiv preprint <a href="https://arxiv.org/abs/2404.01076">https://arxiv.org/abs/2404.01076</a>>

Deville, J. C., and Särndal, C. E. (1992). Calibration estimators in survey sampling. Journal of the American statistical Association, 87(418), 376-382.

#### Examples

```
set.seed(11)
N = 10000
x = data.frame(x1 = rnorm(N, 2, 1), x2= runif(N, 0, 4))
pi = pt((-x[,1] / 2 - x[,2] / 2), 3);
pi = ifelse(pi >.7, .7, pi)
delta = rbinom(N, 1, pi)
Index_S = (delta == 1)
pi_S = pi[Index_S]; d_S = 1 / pi_S
x_S = x[Index_S,]
# Deville & Sarndal(1992)'s calibration using divergence
w1 <- GECal::GEcalib(~ ., dweight = d_S, data = x_S,</pre>
                    const = colSums(cbind(1, x)),
                    entropy = "ET", method = "DS")$w
# Generalized entropy calibration without debiasing covariate
w2 <- GECal::GEcalib(~ ., dweight = d_S, data = x_S,</pre>
                    const = colSums(cbind(1, x)),
                    entropy = "ET", method = "GEC0")$w
all.equal(w1, w2)
# Generalized entropy calibration with debiasing covariate
w3 <- GECal::GEcalib(~ . + g(d_S), dweight = d_S, data = x_S,
                    const = colSums(cbind(1, x, log(1 / pi))),
                    entropy = "ET", method = "GEC")$w
# Generalized entropy calibration with debiasing covariate
# when its population total is unknown
w4 <- GECal::GEcalib(~ . + g(d_S), dweight = d_S, data = x_S,
                    const = colSums(cbind(1, x, NA)),
                    entropy = "ET", method = "GEC")$w
all.equal(w1, w4)
w5 <- GECal::GEcalib(~ . + g(d_S), dweight = d_S, data = x_S,
const = colSums(cbind(1, x, NA)),
entropy = "ET", method = "GEC", K_alpha = "log")$w
```

Synthetic pesticides data in Iowa

# IAdata

## Description

A synthetic proprietary pesticide usage survey data in Iowa CRD(Crop Reporting District) collected from GfK Kynetec in 2020.

#### Format

A data frame with 1197 rows on the following 32 variables:

- Corn10, Corn20, Corn30, Corn40, Corn50, Corn60, Corn70 Haversted acres of corn in each CRD
- Soybean10, Soybean20, Soybean30, Soybean40, Soybean50, Soybean60, Soybean70, Soybean90 Haversted acres of soybean in each CRD
- Alfalfa10, Alfalfa30, Alfalfa40, Alfalfa50, Alfalfa70, Alfalfa80 Haversted acres of alfalfa in each CRD
- Pasture10, Pasture20, Pasture30, Pasture40, Pasture50, Pasture60, Pasture70, Pasture80, Pasture90 Acres of pasture in each CRD
- d Design weights, or inverse first-order inclusion probabilities of the sample
- y Pesticide usage(\$) which is of an interest.

#### Details

The original data is contaminated by adding noise and creating missing values and imputation.

#### Examples

```
data(IAdata)
data(IApimat)
total <- c(
Corn10 = 2093000, Corn20 = 1993600, Corn30 = 1803200, Corn40 = 2084600,
Corn50 = 2056600, Corn60 = 1429400, Corn70 = 2539600,
Soybean10 = 1472980, Soybean20 = 1192860, Soybean30 = 721920,
Soybean40 = 1477680, Soybean50 = 1353600, Soybean60 = 918380,
Soybean70 = 1485200, Soybean90 = 777380, Alfalfa10 = 60590,
Alfalfa30 = 154395, Alfalfa40 = 57816, Alfalfa50 = 150453,
Alfalfa70 = 66065, Alfalfa80 = 240681, Pasture10 = 141947,
Pasture20 = 61476, Pasture30 = 188310, Pasture40 = 213635,
Pasture50 = 160737, Pasture60 = 222214, Pasture70 = 250807,
Pasture80 = 570647, Pasture90 = 232630
)
calibration <- GECal::GEcalib(~ 0, dweight = d, data = IAdata,</pre>
                              const = numeric(0),
                              entropy = "EL", method = "DS")
GECal::estimate(y ~ 1, data = IAdata, calibration = calibration, pimat = IApimat)$estimate
calibration <- GECal::GEcalib(~ 0 + . -y -d, dweight = d, data = IAdata,
                              const = total.
                              entropy = "SL", method = "DS")
GECal::estimate(y ~ 1, data = IAdata, calibration = calibration, pimat = IApimat)$estimate
```

```
IApimat
```

A matrix used for variance estimation in IAdata

#### Description

The matrix that is used for variance estimation in IAdata. The sample is collected from a straitified random sampling.

#### Examples

data(IApimat)

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