Distributed-Lag Structural Equation Modelling with the R package dlsem

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Contents

1 Introduction

Package `dlsem` implements estimation and path analysis functionalities for structural equation models with second-order polynomial lag shapes. In this vignette, the theory on distributed-lag structural equation modelling is presented in Section 2, then the practical use of `dlsem` is illustrated through a worked example in Section 3. Concluding remarks are pointed out in Section 4.

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2 Theory

Distributed-lag structural equation modelling was firstly formalised by [7] as a combination of structural equation modelling (for example, [4]) and distributed-lag linear regression [1]. In this chapter, theory on structural equation modelling and distributed-lag linear regression is briefly reported before presenting distributed-lag structural equation modelling.

2.1 Structural equation modelling

Structural equation modelling (SEM) has a long history starting with the contribution of Wright [9]. The main idea behind SEM is to perform a quantitative assessment of dependence relationships among a set of variables. The basic feature of SEM is a directed acyclic graph (DAG). In a DAG, variables are represented by nodes and directed edges may connect pairs of variables without creating directed cycles (See Figure 1). If a variable receives an edge from another variable, the latter is called *parent* of the former. A DAG encodes a factorization of the joint probability distribution:

$$p(V_1, \ldots, V_m) = \prod_{j=1}^{m} p(V_j \mid \Pi_j) \tag{1}$$

where Π_j is the set of parents of variable V_j. As such, if some pairs of variables are not connected by an edge, the DAG implies a set of conditional independence statements [5]. The DAG may eventually have a causal interpretation. If this is the case, edges represent direct causal relationships. SEM is implemented by simultaneously applying linear regression models:

$$\begin{cases} V_1 = f_1(\Pi_1) \\ \ldots \\ V_j = f_j(\Pi_j) \\ \ldots \\ V_m = f_m(\Pi_m) \end{cases} \tag{2}$$

where $V_j = f_j(\Pi_j)$ is the equation describing the linear regression model where V_j is the response variable and its parents in the DAG are the covariates. A comprehensive review of SEM can be found in [4].

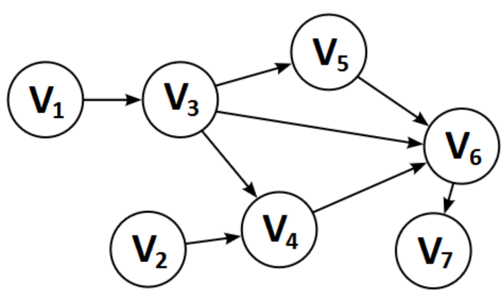

Figure 1: A directed acyclic graph.

An important utility of SEM is path analysis, that is the decomposition of the causal effect of any variable on another. Path analysis can be performed according to trace rules developed by [9] (see also [8]):

- the causal effect associated to each edge in the DAG is represented by the coefficient of the variable originating the edge in the regression model of the variable receiving the edge;
- the causal effect associated to a directed path is represented by the product of the causal effects associated to each edge in the path;
- the causal effect of a variable on another is represented by the sum of the causal effects associated to each directed path connecting the two variables.

For instance, consider variables V_3 and V_6 in the DAG displayed in Figure 1. The directed paths connecting the two variables are (V_3, V_6), (V_3, V_4, V_6) and (V_3, V_5, V_6). The causal effect associated to the first path is the coefficient of V_3 in the regression model of V_6, say $\beta_{6|3}$. The causal effect associated to the second path is the coefficient of V_3 in the regression model of V_4, multiplied by the coefficient of V_4 in the regression model of V_6, say $\beta_{4|3} \cdot \beta_{6|4}$. The causal effect associated to the third path is the coefficient of V_3 in the regression model of V_5 multiplied by the coefficient of V_5 in the regression model of V_6, say $\beta_{5|3} \cdot \beta_{6|5}$. The overal causal effect of V_3 on V_6 is the sum of the causal effect associated to each of the three paths above, say $\beta_{6|3} + \beta_{4|3} \cdot \beta_{6|4} + \beta_{5|3} \cdot \beta_{6|5}$.

Often, the causal effect of a variable on another is termed *overall* causal effect, the causal effect associated to a directed path made by a single edge is called *direct* effect, while the causal effects associated to the other directed paths are denoted as *indirect* effects. In the example above, $\beta_{6|3}$ represents the direct effect of V_3 on V_6, while causal effects $\beta_{4|3} \cdot \beta_{6|4}$ and $\beta_{5|3} \cdot \beta_{6|5}$ represent the indirect effects of V_3 on V_6, and $\beta_{6|3} + \beta_{4|3} \cdot \beta_{6|4} + \beta_{5|3} \cdot \beta_{6|5}$ is the overall causal effect of V_3 on V_6.

2.2 Distributed-lag linear regression

Distributed-lag linear regression is an extension of the classic linear model including lagged instances of one or more quantitative covariates:

$$y_t = \beta_0 + \sum_{j=1}^{J} \sum_{l=0}^{L_j} \beta_{j,l}\ x_{j,t-l} + \epsilon_t \qquad \epsilon_t \sim \mathrm{N}(0, \sigma^2) \tag{3}$$

where y_t is the value of the response variable at time t and $x_{j,t-l}$ is the value of the j-th covariate at l time lags before t. The set $(\beta_{j,0}, \beta_{j,1}, \dots, \beta_{j,L_j})$ is denoted as the *lag shape* of the j-th covariate and represents its effect on the response variable at different time lags. Estimation of a distributed-lag linear regression model using ordinary least squares is inefficient because lagged instances of the same covariate are typically highly correlated. Also, the lag shape of a covariate is completely unrestricted, thus problems of interpretation may arise.

Second-order polynomial lag shapes can be used to solve these drawbacks. They include the endpoint-constrained quadratic lag shape:

$$\beta_{j,l} = \begin{cases} \theta_j \left[-\frac{4}{(b_j - a_j + 2)^2} l^2 + \frac{4(a_j + b_j)}{(b_j - a_j + 2)^2} l - \frac{4(a_j - 1)(b_j + 1)}{(b_j - a_j + 2)^2} \right] & a_j \leq l \leq b_j \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

and the quadratic decreasing lag shape:

$$\beta_{j,l} = \begin{cases} \theta_j \frac{l^2 - 2b_j l + b_j^2}{(b_j - a_j)^2} & a_j \leq l \leq b_j \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

(see Figure 2). The endpoint-constrained quadratic lag shape is zero for a lag $l \leq a_j - 1$ or $l \geq b_j + 1$, and symmetric with mode equal to θ_j at $(a_j + b_j)/2$. The quadratic decreasing lag shape decreases from value θ_j at lag a_j to value 0 at lag b_j according to a quadratic function. We refer to a_j as the *gestation lag*, and to $b_j - a_j$ as the *lag width*. A second-order polynomial lag shape is monotonic in the sign, that is $\beta_{j,l}$ is either non-negative or non-positive for any j and l.

A distributed-lag linear regression model with second-order polynomial lag shapes is linear in parameters θ_j $(j = 1, \dots, J)$, provided that parameters a_j and b_j $(j = 1, \dots, J)$ are known. Thus, one can use ordinary least squares to estimate the parameters of several models where the value of a_j and b_j is varied within a grid of values, and then select the model with the best fit to data. See [1] (Chapter 6) for further details on distributed-lag linear regression.

Note that neither the response variable nor the covariates must contain a trend in order to obtain unbiased estimates [3]. A reasonable procedure is to sequentially apply differentiation to all variables until the Dickey-Fuller test rejects the hypothesis of unit root for all of them.

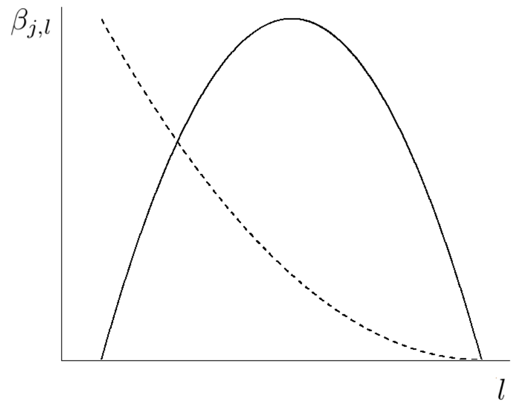

Figure 2: Second-order polynomial lag shapes: endpoint-constrained quadratic lag shape (straight line), quadrating decreasing lag shape (dotted line).

2.3 Distributed-lag structural equation modelling

Distributed-lag structural equation modelling (DLSEM) is an extension of SEM, where variables are related by distributed-lag linear regression models with second-order polynomial lag shapes [7]. In DLSEM, the DAG does not explicitly include time lags but a special semantic holds:

- if an edge connects two variables, there is at least one time lag where the coefficient of the variable originating the edge in the regression model of the variable receiving the edge is non-zero.

DLSEM can be used to perform path analysis at different time lags by extending tracing rules reported in Subsection 2.1 (see the box below).

Tracing rules for DLSEM

- The causal effect associated to each edge in the DAG at lag k is represented by the coefficient at lag k of the variable originating the edge in the regression model of the variable receiving the edge.
- The causal effect associated to a directed path at lag k is computed as follows:
  1. denote the number of edges in the path as p;
  2. enumerate all the possible p-uples of lags, one lag for each of the p edges, such that their sum is equal to k;
  3. for each p-uple of lags:
     - for each lag in the p-uple, compute the coefficient associated to the corresponding edge at that lag;
     - compute the product of all these coefficients;
  4. sum all these products.
- The causal effect of a variable on another at lag k is represented by the sum of the causal effects at lag k associated to each directed path connecting the two variables.

A causal effect evaluated at a single lag is denoted as *instantaneous* causal effect. The *cumulative* causal effect at a prespecified lag, say k, is obtained by summing all the instantaneous causal effects for each lag up to k.

Parameter estimation in DLSEM can be performed by applying the method proposed in Subsection 2.2 to each regression model. An edge of the DAG is considered as statistically significant if there is at least one time lag where the estimate of the coefficient of the variable originating the edge in the regression model of the variable receiving the edge is statistically significant.

3 Distributed-lag structural equation modelling with dlsem

The practical use of package `dlsem` is illustrated by an application to impact assessment of research activity on European Agriculture. It is widely acknowledged that research activity is effective in increasing productivity, however, it is also expected to improve profitability and consumer surplus independently from productivity. Here, a distributed-lag structural equation model with DAG shown in Figure 3 is estimated from dataset `agres` in order to test whether the influence through time of research activity on profitability and consumer surplus is direct and/or mediated by productivity.

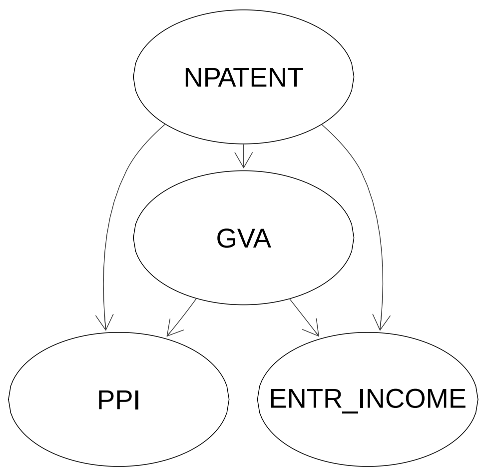

Figure 3: The hypothesized DAG for impact assessment of research activity on European Agriculture. 'RES': research activity. 'PROD': productivity. 'PROFIT': profitability. 'C_SURPL': consumer surplus.

Dataset `agres` contains data for 10 European countries (Austria, Germany, Spain, Finland, France, Ireland, Italy, Netherlands, Sweden, United Kingdom) in the period 1990-2010 from the EUROSTAT database (`http://ec.europa.eu/eurostat/data/database`).

```
> data(agres)
> summary(agres)
```

```
    COUNTRY          YEAR            GDP              FARM_SIZE
 AT      : 22   Min.   :1991   Min.   :  85220   Min.   :0.01820
 BE      : 22   1st Qu.:1996   1st Qu.: 218183   1st Qu.:0.03370
 DE      : 22   Median :2002   Median : 356676   Median :0.05104
 DK      : 22   Mean   :2002   Mean   : 879657   Mean   :0.06222
 EL      : 22   3rd Qu.:2007   3rd Qu.:1678138   3rd Qu.:0.07544
 ES      : 22   Max.   :2012   Max.   :3158590   Max.   :0.21481
 (Other):176
    NPATENT          GVA              PPI             ENTR_INCOME
 Min.   :  0.04   Min.   :  968   Min.   : 60.36   Min.   : 18.75
 1st Qu.:  7.75   1st Qu.: 3593   1st Qu.: 97.14   1st Qu.: 70.70
 Median : 24.18   Median : 6782   Median :102.07   Median : 87.80
 Mean   : 55.27   Mean   :13471   Mean   :105.52   Mean   : 91.85
```

```
 3rd Qu.: 71.73   3rd Qu.:21024   3rd Qu.:111.12   3rd Qu.:107.44
 Max.   :472.09   Max.   :41048   Max.   :191.60   Max.   :229.36
 NA's   :1                        NA's   :9        NA's   :8
```

Variable NPATENT representing the number of Agriculture-related patent applications will be used as a proxy of research activity in Agriculture. Variable GVA representing the gross value added of Agriculture will be used as a proxy of agricultural productivity. Variable ENTR_INCOME representing the net entrepreneurial income index will be used as a proxy of profitability. Variable PPI representing the price index of agricultural products will be used as a proxy of consumer surplus.

3.1 The model code

The first step is the specification of the model code containing the hypothesized DAG and the lag shapes. The model code must be a list of formulas, one for each regression model. In each formula, the response and the covariates must be quantitative variables and operators quec( ) and qdec( ) can be employed to specify, respectively, an endpoint-constrained quadratic or a quadratic decreasing lag shape. Each of these operators has three arguments: the name of the variable to which the lag shape is applied, the minimum lag with a non-zero coefficient (a_j), and the maximum lag with a non-zero coefficient (b_j). If none of these two operators is applied to a variable, it is assumed that the coefficient associated to that variable is 0 for time lags greater than 0 (no lag shape). The group factor and context variables must not be specified in the model code (see Subsection 3.3). The regression model for variables with no parents besides the group factor and context variables can be omitted from the model code. In this illustration, we assume an endpoint-constrained quadratic lag shape between 0 and 15 time lags for all variables:

```
> mycode <- list(
+   GVA~quec(NPATENT,0,15),
+   PPI~quec(NPATENT,0,15)+quec(GVA,0,15),
+   ENTR_INCOME~quec(NPATENT,0,15)+quec(GVA,0,15)
+   )
```

3.2 Control options

The second step is the specification of control options. Control options must be a named list containing one or more among several components. The key component is adapt, a named vector of logical values where each value must refer to one response variable and indicates whether values a_j and b_j for each lag shape in the regression model of that variable must be selected on the basis of the best fit to data, instead of employing the ones specified in the model code. If adaption is requested for a regression model, three further components are taken into account: max.gestation, min.width and sign. Each of these three components is a named list, where each component of the list must refer to one response variable and must be a named vector including, respectively, the maximum gestation lag, the minimum lag width and the sign (either '+' for non-negative, or '-' for non-positive) of the coefficients of one or more covariates. In this illustration, we choose to perform adaptation of lag shapes for all regression models with the following constraints: (i) maximum gestation lag of 3 years, (ii) minimum lag width of 5 years, (iii) all coefficients with non-negative sign, excepting the ones in the regression model of the price index of agricultural products, as consumer surplus improves with the decreasing of prices:

```
> mycontrol <- list(
+   adapt=c(GVA=T,PPI=T,ENTR_INCOME=T),
+   max.gestation=list(GVA=c(NPATENT=3),PPI=c(NPATENT=3,GVA=3),
+     ENTR_INCOME=c(NPATENT=3,GVA=3)),
+   min.width=list(GVA=c(NPATENT=5),PPI=c(NPATENT=5,GVA=5),
+     ENTR_INCOME=c(NPATENT=5,GVA=5)),
+   sign=list(GVA=c(NPATENT="+"),PPI=c(NPATENT="-",GVA="-"),
```

```
+     ENTR_INCOME=c(NPATENT="+",GVA="+"))
+   )
```

3.3 Estimation

Once the model code and control options are specified, the structural model can be estimated from data using the command dlsem( ). The user can indicate a group factor to argument group and one or more context variables to argument context. By indicating the group factor, one intercept for each level of the group factor will be estimated in each regression model. By indicating context variables, they will be included as covariates in each regression model in order to eliminate spurious effects due to differences between the levels of the group factor. Each context variable can be either qualitative or quantitative and its coefficient in each regression model is 0 for time lags greater than 0 (no lag shape). Furthermore, the user can decide to perform any number of the following operations:

- differentiation until the hypothesis of unit root is rejected by the Dickey-Fuller test for all the quantitative variables (by setting argument uniroot.check to TRUE);
- imputation of missing values for quantitative variables using the Expectation-Maximization algorithm [2] (by setting argument imputation to TRUE);
- apply the logarithmic transformation to all quantitative variables in order to interpret each coefficient as an elasticity (by setting argument log to TRUE).

In this illustration, we indicate the country as the group factor, gross domestic product and average farm size as context variables, allow differentiation until stationarity, imputation of missing values and logarithmic transformation for all quantitative variables:

```
> mod0 <- dlsem(mycode,group="COUNTRY",context=c("GDP","FARM_SIZE"),
+   data=agres,control=mycontrol,uniroot.check=T,imputation=T,log=T)
```

```
Checking stationarity...
 Order 1 differentiation performed
Starting EM...
EM iteration 1. Log-likelihood: 1394.8399
EM iteration 2. Log-likelihood: 1395.3395
EM iteration 3. Log-likelihood: 1395.4016
EM iteration 4. Log-likelihood: 1395.4073
EM iteration 5. Log-likelihood: 1395.4067
EM converged after 4 iterations. Log-likelihood: 1395.4067
Start estimation...
 Estimating regression model 1/4 (NPATENT)
 Estimating regression model 2/4 (GVA)
 Estimating regression model 3/4 (PPI)
 Estimating regression model 4/4 (ENTR_INCOME)
Estimation completed
```

After estimating the structural model, the user can display the DAG including only statistically significant edges.

```
> plot(mod0)
```

The result is shown in Figure 4: each edge is coloured according to the sign of its causal effect (green for non-negative, red for non-positive), while the group factor and context variables are omitted from the DAG.

We see that all edges are statistically significant, excepting the one linking research activity to profitability. This provides evidence that the effect of research activity on consumer surplus is both direct and mediated by productivity, and the effect of research activity on profitability is only mediated by productivity.

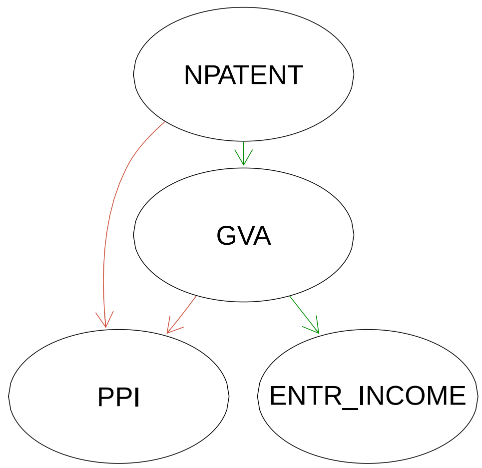

Figure 4: The DAG including only statistically significant edges. Green: non-negative causal effect. Red: non-positive causal effect.

The user can also request the summary of estimation:

```
> summary(mod0)
$NPATENT

Call:
"NPATENT ~ COUNTRY+GDP+FARM_SIZE"

Residuals:
    Min      1Q  Median      3Q     Max 
-3.6255 -0.2156  0.0172  0.2146  3.8613 

Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
factor(COUNTRY)AT -0.032677   0.153155  -0.213    0.831
factor(COUNTRY)BE -0.051062   0.153745  -0.332    0.740
factor(COUNTRY)DE -0.029085   0.153605  -0.189    0.850
factor(COUNTRY)DK -0.028752   0.153359  -0.187    0.851
factor(COUNTRY)EL  0.008343   0.151246   0.055    0.956
factor(COUNTRY)ES -0.033427   0.154347  -0.217    0.829
factor(COUNTRY)FI -0.012809   0.155401  -0.082    0.934
factor(COUNTRY)FR -0.061953   0.152594  -0.406    0.685
factor(COUNTRY)IE -0.080913   0.167465  -0.483    0.629
factor(COUNTRY)IT  0.001801   0.151346   0.012    0.991
factor(COUNTRY)NL -0.063467   0.153436  -0.414    0.679
factor(COUNTRY)PT  0.028596   0.151790   0.188    0.851
factor(COUNTRY)SE -0.093923   0.154125  -0.609    0.543
factor(COUNTRY)UK -0.102351   0.154367  -0.663    0.508
GDP                2.060751   1.586265   1.299    0.195
FARM_SIZE          0.049937   0.562659   0.089    0.929

Residual standard error: 0.686 on 278 degrees of freedom
  (14 observations deleted due to missingness)
Multiple R-squared:  0.008403,	Adjusted R-squared:  -0.04867 
F-statistic: 0.1472 on 16 and 278 DF,  p-value: 1


$GVA

Call:
```

```
"GVA ~ COUNTRY+quec(NPATENT,1,15)+GDP+FARM_SIZE"

Residuals:
      Min        1Q    Median        3Q       Max
-0.298977 -0.034302  0.000572  0.041155  0.257996

Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
factor(COUNTRY)AT   -7.015e-02  5.340e-02  -1.314   0.1935
factor(COUNTRY)BE   -6.750e-02  4.757e-02  -1.419   0.1605
factor(COUNTRY)DE   -2.994e-02  4.272e-02  -0.701   0.4858
factor(COUNTRY)DK   -2.912e-02  3.948e-02  -0.737   0.4634
factor(COUNTRY)EL   -1.265e-01  6.798e-02  -1.860   0.0672 .
factor(COUNTRY)ES   -1.297e-01  6.765e-02  -1.917   0.0595 .
factor(COUNTRY)FI   -3.056e-02  4.268e-02  -0.716   0.4765
factor(COUNTRY)FR   -1.918e-02  3.789e-02  -0.506   0.6145
factor(COUNTRY)IE   -8.036e-02  4.204e-02  -1.912   0.0602 .
factor(COUNTRY)IT   -5.455e-02  4.506e-02  -1.210   0.2303
factor(COUNTRY)NL   -2.338e-02  3.948e-02  -0.592   0.5557
factor(COUNTRY)PT   -1.879e-01  9.417e-02  -1.995   0.0501 .
factor(COUNTRY)SE   -4.723e-02  3.990e-02  -1.184   0.2406
factor(COUNTRY)UK    4.418e-05  3.602e-02   0.001   0.9990
theta0_quec.NPATENT  1.015e-01  4.750e-02   2.137   0.0362 *
GDP                  2.555e-01  3.358e-01   0.761   0.4494
FARM_SIZE            1.438e-01  1.372e-01   1.048   0.2982
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.08788 on 67 degrees of freedom
  (224 observations deleted due to missingness)
Multiple R-squared:  0.1184,        Adjusted R-squared:  -0.1052
F-statistic: 0.5295 on 17 and 67 DF,  p-value: 0.9282


$PPI

Call:
"PPI ~ COUNTRY+quec(NPATENT,0,13)+quec(GVA,0,14)+GDP+FARM_SIZE"

Residuals:
      Min        1Q    Median        3Q       Max
-0.167506 -0.036284 -0.000584  0.045151  0.132116

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
factor(COUNTRY)AT    0.09617    0.02959   3.250  0.00169 **
factor(COUNTRY)BE    0.07313    0.02898   2.524  0.01360 *
factor(COUNTRY)DE    0.05803    0.02540   2.285  0.02499 *
factor(COUNTRY)DK    0.08315    0.02528   3.290  0.00149 **
factor(COUNTRY)EL    0.08880    0.03868   2.296  0.02432 *
factor(COUNTRY)ES    0.09384    0.03102   3.025  0.00334 **
factor(COUNTRY)FI    0.07735    0.02576   3.002  0.00357 **
factor(COUNTRY)FR    0.06450    0.02533   2.546  0.01281 *
factor(COUNTRY)IE   -0.01945    0.04607  -0.422  0.67398
factor(COUNTRY)IT    0.08050    0.02574   3.128  0.00245 **
factor(COUNTRY)NL    0.03607    0.02562   1.408  0.16303
factor(COUNTRY)PT    0.13945    0.04462   3.125  0.00248 **
factor(COUNTRY)SE    0.05435    0.02754   1.973  0.05193 .
factor(COUNTRY)UK    0.07131    0.02428   2.938  0.00432 **
theta0_quec.NPATENT -0.07098    0.02161  -3.285  0.00152 **
theta0_quec.GVA     -0.17540    0.07322  -2.395  0.01893 *
GDP                  2.04719    0.22917   8.933 1.19e-13 ***
FARM_SIZE            0.14364    0.09643   1.490  0.14027
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 0.06331 on 80 degrees of freedom
  (210 observations deleted due to missingness)
Multiple R-squared:  0.641,	Adjusted R-squared:  0.5602
F-statistic: 7.934 on 18 and 80 DF,  p-value: 1.936e-11


$ENTR_INCOME

Call:
"ENTR_INCOME ~ COUNTRY+quec(NPATENT,1,13)+quec(GVA,1,14)+GDP+FARM_SIZE"

Residuals:
     Min       1Q   Median       3Q      Max
-0.96399 -0.12989  0.00663  0.14634  0.56581

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
factor(COUNTRY)AT   -0.14959    0.13458  -1.112 0.269665
factor(COUNTRY)BE   -0.26243    0.13144  -1.997 0.049281 *
factor(COUNTRY)DE   -0.13999    0.11580  -1.209 0.230280
factor(COUNTRY)DK   -0.39852    0.11405  -3.494 0.000778 ***
factor(COUNTRY)EL   -0.07998    0.16902  -0.473 0.637349
factor(COUNTRY)ES   -0.24574    0.14855  -1.654 0.101998
factor(COUNTRY)FI   -0.09529    0.11379  -0.837 0.404825
factor(COUNTRY)FR   -0.12296    0.11267  -1.091 0.278418
factor(COUNTRY)IE    0.17533    0.16480   1.064 0.290585
factor(COUNTRY)IT   -0.06445    0.11775  -0.547 0.585646
factor(COUNTRY)NL   -0.10808    0.11422  -0.946 0.346867
factor(COUNTRY)PT   -0.24381    0.21085  -1.156 0.250994
factor(COUNTRY)SE   -0.08117    0.11962  -0.679 0.499352
factor(COUNTRY)UK   -0.09867    0.10783  -0.915 0.362895
theta0_quec.NPATENT  0.16322    0.10498   1.555 0.123936
theta0_quec.GVA      0.62290    0.29551   2.108 0.038173 *
GDP                 -3.01030    1.01618  -2.962 0.004018 **
FARM_SIZE           -1.21328    0.42983  -2.823 0.006007 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2827 on 80 degrees of freedom
  (210 observations deleted due to missingness)
Multiple R-squared:  0.2983,	Adjusted R-squared:  0.1404
F-statistic: 1.889 on 18 and 80 DF,  p-value: 0.02853
```

The summary of estimation returns estimates of parameters θ_j ($j = 1, \ldots, J$). Instead, the command edgeCoeff( ) can be used to obtain estimates and confidence intervals of coefficients at the relevant time lags $\beta_{j,l}$ ($j = 1, \ldots, J$; $l = 0, 1, \ldots$):

```
> edgeCoeff(mod0)
$`0`
                           2.5%         50%        97.5%
GVA~NPATENT          0.00000000  0.00000000  0.000000000
PPI~NPATENT         -0.02820862 -0.01766653 -0.007124428
PPI~GVA             -0.07474607 -0.04111016 -0.007474252
ENTR_INCOME~NPATENT  0.00000000  0.00000000  0.000000000
ENTR_INCOME~GVA      0.00000000  0.00000000  0.000000000

$`1`
                            2.5%         50%       97.5%
GVA~NPATENT          0.001971873  0.02379354  0.04561522
PPI~NPATENT         -0.052387442 -0.03280926 -0.01323108
PPI~GVA             -0.139526002 -0.07673897 -0.01395194
ENTR_INCOME~NPATENT  0.000000000  0.00000000  0.00000000
ENTR_INCOME~GVA      0.010878743  0.15503301  0.29918727
```

```
$`2`
                          2.5%         50%       97.5%
GVA~NPATENT          0.00368083  0.04441462  0.08514840
PPI~NPATENT         -0.07253646 -0.04542821 -0.01831996
PPI~GVA             -0.19433979 -0.10688642 -0.01943306
ENTR_INCOME~NPATENT  0.00000000  0.00000000  0.00000000
ENTR_INCOME~GVA      0.02020338  0.28791844  0.55563350

$`3`
                          2.5%         50%       97.5%
GVA~NPATENT          0.00512687  0.06186322  0.11859956
PPI~NPATENT         -0.08865567 -0.05552337 -0.02239106
PPI~GVA             -0.23918743 -0.13155252 -0.02391761
ENTR_INCOME~NPATENT  0.00000000  0.00000000  0.00000000
ENTR_INCOME~GVA      0.02797391  0.39865630  0.76933869

$`4`
                           2.5%         50%       97.5%
GVA~NPATENT          0.006309994  0.07613934  0.14596869
PPI~NPATENT         -0.100745081 -0.06309473 -0.02544439
PPI~GVA             -0.274068932 -0.15073726 -0.02740559
ENTR_INCOME~NPATENT  0.000000000  0.00000000  0.00000000
ENTR_INCOME~GVA      0.034190336  0.48724659  0.94030285

$`5`
                           2.5%         50%       97.5%
GVA~NPATENT          0.007230202  0.08724300  0.16725579
PPI~NPATENT         -0.108804688 -0.06814231 -0.02747994
PPI~GVA             -0.298984290 -0.16444065 -0.02989701
ENTR_INCOME~NPATENT  0.000000000  0.00000000  0.00000000
ENTR_INCOME~GVA      0.038852654  0.55368931  1.06852596

$`6`
                           2.5%         50%       97.5%
GVA~NPATENT          0.007887493  0.09517418  0.18246087
PPI~NPATENT         -0.112834491 -0.07066610 -0.02849771
PPI~GVA             -0.313933504 -0.17266268 -0.03139186
ENTR_INCOME~NPATENT  0.000000000  0.00000000  0.00000000
ENTR_INCOME~GVA      0.041960866  0.59798445  1.15400804

$`7`
                           2.5%         50%       97.5%
GVA~NPATENT          0.008281867  0.09993289  0.19158391
PPI~NPATENT         -0.112834491 -0.07066610 -0.02849771
PPI~GVA             -0.318916576 -0.17540336 -0.03189014
ENTR_INCOME~NPATENT  0.000000000  0.00000000  0.00000000
ENTR_INCOME~GVA      0.043514972  0.62013203  1.19674908

$`8`
                           2.5%         50%       97.5%
GVA~NPATENT          0.008413325  0.10151913  0.19462492
PPI~NPATENT         -0.108804688 -0.06814231 -0.02747994
PPI~GVA             -0.313933504 -0.17266268 -0.03139186
ENTR_INCOME~NPATENT  0.000000000  0.00000000  0.00000000
ENTR_INCOME~GVA      0.043514972  0.62013203  1.19674908

$`9`
                           2.5%         50%       97.5%
GVA~NPATENT          0.008281867  0.09993289  0.19158391
PPI~NPATENT         -0.100745081 -0.06309473 -0.02544439
PPI~GVA             -0.298984290 -0.16444065 -0.02989701
ENTR_INCOME~NPATENT  0.000000000  0.00000000  0.00000000
ENTR_INCOME~GVA      0.041960866  0.59798445  1.15400804
```

```
$`10`
                          2.5%         50%       97.5%
GVA~NPATENT          0.007887493  0.09517418  0.18246087
PPI~NPATENT         -0.088655672 -0.05552337 -0.02239106
PPI~GVA             -0.274068932 -0.15073726 -0.02740559
ENTR_INCOME~NPATENT  0.000000000  0.00000000  0.00000000
ENTR_INCOME~GVA      0.038852654  0.55368931  1.06852596

$`11`
                          2.5%         50%       97.5%
GVA~NPATENT          0.007230202  0.08724300  0.16725579
PPI~NPATENT         -0.072536459 -0.04542821 -0.01831996
PPI~GVA             -0.239187432 -0.13155252 -0.02391761
ENTR_INCOME~NPATENT  0.000000000  0.00000000  0.00000000
ENTR_INCOME~GVA      0.034190336  0.48724659  0.94030285

$`12`
                          2.5%         50%       97.5%
GVA~NPATENT          0.006309994  0.07613934  0.14596869
PPI~NPATENT         -0.052387442 -0.03280926 -0.01323108
PPI~GVA             -0.194339788 -0.10688642 -0.01943306
ENTR_INCOME~NPATENT  0.000000000  0.00000000  0.00000000
ENTR_INCOME~GVA      0.027973911  0.39865630  0.76933869

$`13`
                         2.5%         50%        97.5%
GVA~NPATENT          0.00512687  0.06186322  0.118599563
PPI~NPATENT         -0.02820862 -0.01766653 -0.007124428
PPI~GVA             -0.13952600 -0.07673897 -0.013951937
ENTR_INCOME~NPATENT  0.00000000  0.00000000  0.000000000
ENTR_INCOME~GVA      0.02020338  0.28791844  0.555633502

$`14`
                         2.5%         50%        97.5%
GVA~NPATENT          0.00368083  0.04441462  0.085148405
PPI~NPATENT          0.00000000  0.00000000  0.000000000
PPI~GVA             -0.07474607 -0.04111016 -0.007474252
ENTR_INCOME~NPATENT  0.00000000  0.00000000  0.000000000
ENTR_INCOME~GVA      0.01087874  0.15503301  0.299187270

$`15`
                         2.5%        50%      97.5%
GVA~NPATENT         0.001971873 0.02379354 0.04561522
PPI~NPATENT         0.000000000 0.00000000 0.00000000
PPI~GVA             0.000000000 0.00000000 0.00000000
ENTR_INCOME~NPATENT 0.000000000 0.00000000 0.00000000
ENTR_INCOME~GVA     0.000000000 0.00000000 0.00000000
```

3.4 Path analysis

Path analysis can be performed using the command pathAnal( ). The user must specify one or more starting variables (argument from) and the ending variable (argument to). Optionally, specific time lags on which path analysis should be focused can be provided to argument lag, otherwise all the relevant ones are considered. Also, the user can choose whether instantaneous (argument cumul set to FALSE, the default) or cumulative (argument cumul set to TRUE) causal effects must be returned. Here we perform two path analysis tasks: one from research activity to profitability and the other from research activity to consumer surplus. For both, we focus on time lags 5, 10, 15, 20 and 25, and request cumulative causal effects:

```
> pathAnal(mod0,from="NPATENT",to="ENTR_INCOME",lag=c(5,10,15,20,25),cumul=T)

$`NPATENT*GVA*ENTR_INCOME`
         2.5%        50%     97.5%
```

```
5  0.02276737 0.1082044 0.1936413
10 0.47814179 1.0839758 1.6898097
15 1.75013573 3.3444982 4.9388607
20 3.02212968 5.6050207 8.1879117
25 3.47750409 6.5807921 9.6840801

$overall
         2.5%       50%     97.5%
5  0.02276737 0.1082044 0.1936413
10 0.47814179 1.0839758 1.6898097
15 1.75013573 3.3444982 4.9388607
20 3.02212968 5.6050207 8.1879117
25 3.47750409 6.5807921 9.6840801
```

```
> pathAnal(mod0,from="NPATENT",to="PPI",lag=c(5,10,15,20,25),cumul=T)
$`NPATENT*GVA*PPI`
          2.5%         50%      97.5%
5  -0.09077204 -0.05435072 -0.0179294
10 -0.59019516 -0.39351888 -0.1968426
15 -1.55081124 -1.08108464 -0.6113580
20 -2.44436338 -1.71655161 -0.9887398
25 -2.84103140 -1.98134075 -1.1216501

$`NPATENT*PPI`
         2.5%        50%      97.5%
5  -0.4513380 -0.2826644 -0.1139909
10 -0.9752124 -0.6107570 -0.2463017
15 -1.1283449 -0.7066610 -0.2849771
20 -1.1283449 -0.7066610 -0.2849771
25 -1.1283449 -0.7066610 -0.2849771

$overall
        2.5%        50%      97.5%
5  -0.542110 -0.3370151 -0.1319203
10 -1.565408 -1.0042759 -0.4431443
15 -2.679156 -1.7877457 -0.8963352
20 -3.572708 -2.4232126 -1.2737170
25 -3.969376 -2.6880018 -1.4066272
```

The output of path analysis is a list of matrices, each containing estimates and confidence intervals of the causal effect associated to each path connecting the starting variables to the ending variable at the requested time lags. Also, estimates and confidence intervals of the overall causal effect is shown in the component named overall.

Since the logarithmic trasformation was applied to all quantitative variables, causal effects above are interpreted as elasticities, that is, for a 1% of patent applications more, profitability and consumer surplus are expected to grow by 6.6% and 1.8%, respectively, after 25 years.

The estimated lag shape associated to an overall causal effect can be displayed using the command lagPlot( ):

```
> lagPlot(mod0,from="NPATENT",to="ENTR_INCOME")
> lagPlot(mod0,from="NPATENT",to="PPI")
```

The result is shown in Figure 5.

4 Concluding remarks

Package `dlsem` is conceived to perform impact analysis, that is the quantitative assessment of the consequences on a system due to an internal or external impulse, using distributed-lag structural

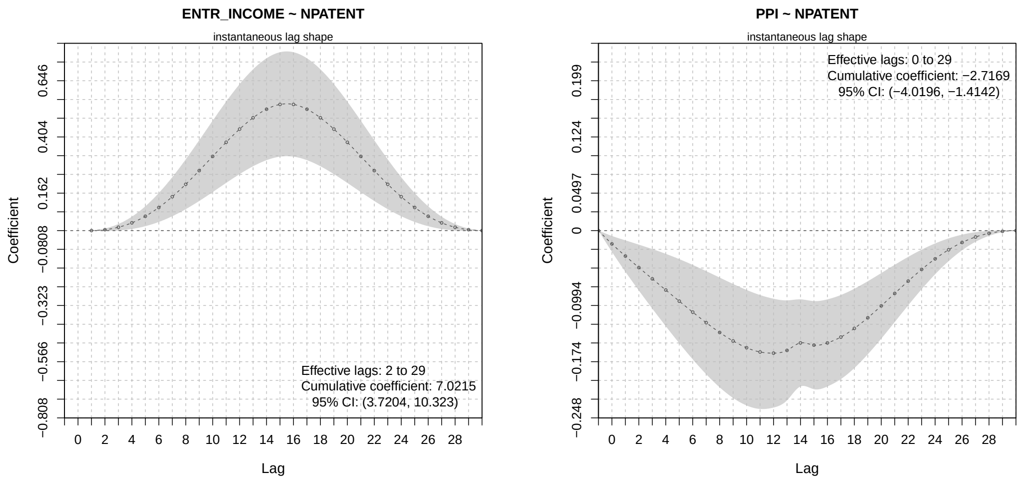

Figure 5: The estimated lag shape associated to the overall causal effect of research activity on profitability and consumer surplus. 95% confidence intervals are shown in grey.

equation modelling with second-order polynomial lag shapes. Second-order polynomial lag shapes have several advantages, including simplicity of estimation and a clear interpretation of parameters for domain experts. The illustration proposed in this tutorial applies impact analysis to a simplified problem of agricultural economics. The model here proposed can be extended by considering research investment as a direct cause of research activity, as well as a larger number variables to better measure the macroeconomic state of the system.

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